PREFACE

In the curricular structure introduced by this University for students of Post-Graduate degree programme, the opportunity to pursue Post-Graduate course in a subject is introduced by this University is equally available to all learners. Instead of being guided by any presumption about ability level, it would perhaps stand to reason if receptivity of a learner is judged in the course of the learning process. That would be entirely in keeping with the objectives of open education which does not believe in artificial differentiation. I am happy to note that university has been recently accredited by National Assessment and Accreditation Council of India (NAAC) with grade 'A'.

Keeping this in view, the study materials of the Post Graduate level in different subjects are being prepared on the basis of a well laid-out syllabus. The course structure combines the best elements in the approved syllabi of Central and State Universities in respective subjects. It has been so designed as to be upgradable with the addition of new information as well as results of fresh thinking and analysis.

The accepted methodology of distance education has been followed in the preparation of these study materials. Co-operation in every form of experienced scholars is indispensable for a work of this kind. We, therefore, owe an enormous debt of gratitude to everyone whose tireless efforts went into the writing, editing, and devising of a proper layout of the materials. Practically speaking, their role amounts to an involvement in 'layout of the materials. Practically speaking, their role amounts to an involvement in 'invisible teaching'. For, whoever makes use of these study materials would virtually derive the benefit of learning under their collective care without each being seen by the other.

The more a learner would seriously pursue these study materials, the easier it will be for him or her to reah out to larger horizons of a subject. Care has also been taken to make the language lucid and presentation attrative so that they may be rated as quality self-learning materials. If anything remains still obscure or difficult to follow, arrangements are there to come to terms with them through the counselling sessions regularly available at the network of study centres set up by the University.

Needless to add, a great deal of these efforts is still experimental—in fact, pioneering in certain areas. Naturally, there is every possibility of some lapse or deficiency here and there. However, these do admit of rectification and further improvement in due course. On the whole, therefore, these study materials are expected to evoke wider appreciation the more they receive serious attention of all concerned.

Professor (Dr.) Subha Sankar Sarkar
Vice-Chancellor

Netaji Subhas Open University

Post Graduate Degree Programme

MA in Economics

Course : Microeconomic Theory

Course Code : PGEC-VI

First Print : February, 2022

Printed in accordance with the regulations of the Distance Education
Bureau of the University Grants Commission.

Netaji Subhas Open University

Post Graduate Degree Programme

MA in Economics

Course : Microeconomic Theory
Course Code : PGEC-VI

: Course Writer :
Sovik Mukherjee
Assistant Professor of Economics,
St. Xavier's University

: Course Editor :
Dr. Asim Karmakar
Assistant Professor of Economics,
Netaji Subhas Open University

Format Editor
Priyanthi Bagchi
Assistant Professor of Economics, NSOU

Netaji Subhas Open University

PG : Economics (PGEC)

PGEC-VI : Microeconomic Theory

Unit 1 ❑ Introduction to Basics of Consumption

Structure

1.1 Objectives

After going through this unit you will be able

- To know about budget line and its properties, also cardinal vs ordinal utility;
- To have an idea about indifference curve and its properties and consumer's optimal bundle; and some exceptional indifference curves;

- To have an idea about Samuelson's formulation of revealed preference theory;
- to get a knowledge about duality approach and its associated indirect utility function, its properties and expenditure function; and
- Learn about Marshallian consumer surplus and its Hicksian attempt to rehabilitate Marshallian consumer surplus.

1.2 Introduction

The postulate of rationality is the customary point of departure in the theory of consumer behavior. The consumer is assumed to choose among the available alternatives in such a manner that the satisfaction derived from consuming commodities is as large as possible. This implies that he is aware of the alternatives. All information pertaining to the satisfaction that the consumer derives from various quantities of commodities is contained in his *utility function.*

The 19th century economists W. Stanley Jevons, Leon Walras and Alfred Marshall considered utility measurable, just as the weight of objects is measurable. But in course of time, concept of indifference curve came to the fore. In this concept, the consumer is able to rank commodities in order of preference. The consumer possesses an *ordinary* utility measure. He maximizes utility subject to his budget constraint. These are discussed in Section 1.3 to 1.5.

A utility function that generates estimable expenditure function is also discussed here. The theory of revealed preference is summarized. Moreover, measures for the Marshallian consumer's surplus and its extension by Hicks through using equivalent and compensating variation are highlighted in Section 1.8.

1.3 Budget Line

The budget line indicates all the combinations of good X and good Y that the consumer can buy if he spends a fixed amount of money at fixed prices for the products. Thus, budget line shows the quantities of goods available to a consumer given money income and prices of goods (Fig 1.1). It is also sometimes called the isocost line because all points on it represent bundles of goods with the same total cost.

It is represented by the equation : **M = PX.X + PY.Y** where, M = budget or money income of the consumer and PX and PY represent the prices of good X and good Y respectively.

Slope of the budget line :

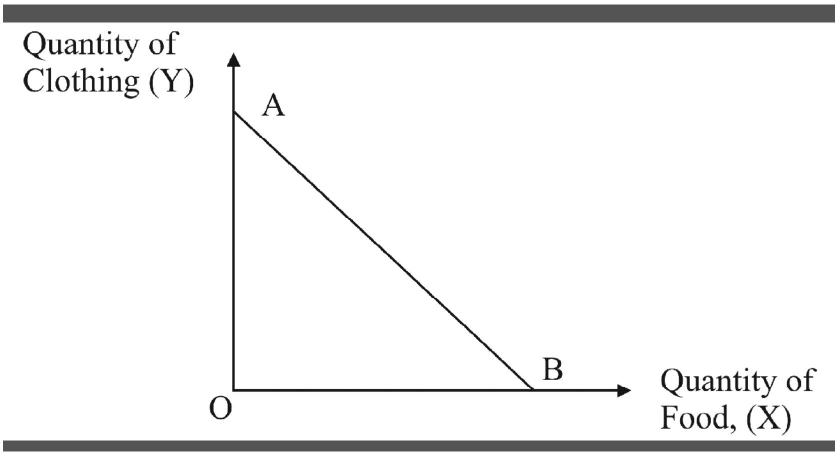
For example, the individual may have Rs. 100 to spend on the two products per week. Hence the equation of the budget line is

100 = PX.X + PY.Y

or, PY.Y = 100 – PX.X

or, $Y = - (Px/Py).X + 100/P_Y$

Hence the slope of the budget line is- (Px/Py) or negative inverse of the ratio of the prices of two commodities X and Y.



1.1 Budget Line

Properties of the Budget Line

1. The Budget line will be a straight line.
2. It will have a negative scope.
3. Its slope will be equal to the negative inverse of the ratio of the prices of the two commodities respectively. i.e., If the equation of the budget line is M = Px.X + Py.Y, then $Y = -(Px/Py).X + 100/P_Y$. Hence slope of the budget line is where Px and Py are the unit prices of X and Y respectively.

4. Suppose two budget lines involve the same commodity prices but represent the expenditure of different amounts of money. Then the two lines will be parallel.

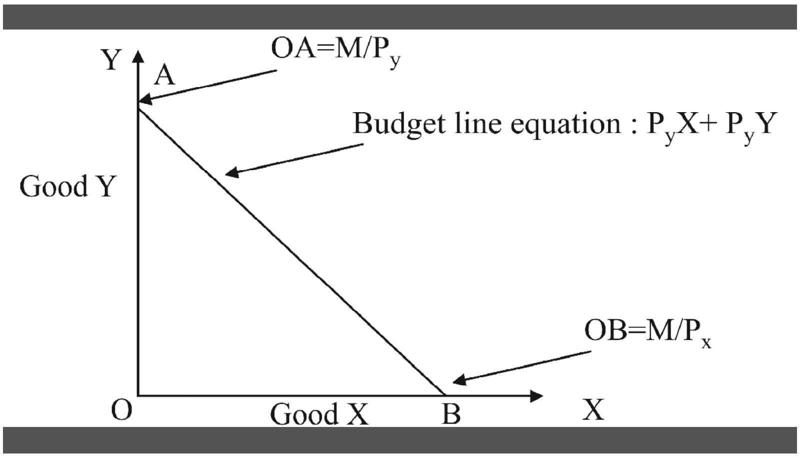


Fig 1.2: Budget Line or Budget Constraint

The knowledge of the concept of budget line or what is also called budget constraint is essential for understanding the theory of consumer's equilibrium.

It is clear from above that budget line graphically shows the budget constraint. The combinations of commodities lying to the right of the budget line are unattainable because income of the consumer is not sufficient to buy those combinations. Given consumer's income and price of the two goods, the combinations of goods lying to the left of the budget line are attainable, that is, the consumer can buy any one of them.

It is also important to remember that the intercept OA on the Y-axis in Figure 1.2 equals the amount of his entire income (M) divided by the price (Py) of commodity Y. This is, OA = . Likewise, the intercept OB on the X-axis measures the total income divided by the price of commodity X. Thus OB = M/Px.

The budget line can be written algebraically as follows :

$$PxX + PyY = M \ldots\ldots\ldots\ldots (1)$$

$$Y = M/Py - (Px/\ Py)\ X$$

where Px and Py denote prices of goods X and Y respectively and M stands for money income :

The above budget line equation (1) implies that, given the money income of the consumer and prices of the two goods, every combination lying on the budget line will cost the same amount of money and can therefore by purchased with the given income. The budget line can be defined **as a set of combinations of two commodities that can be purchased if whole of the given income is spent on them and its slope is equal to the negative inverse of the price ratio of the commodities**.

Budget Space :

It should be carefully understood that the budget equation PxX + PyY = M or Y = M/Py- (Px/Py) depicted by the budget line in Fig. 1.2 only describes the budget line and not the budget space. **A budget space shows a set of all combinations the two commodities that can be purchased by spending the whole or a part of the given income.**

Therefore, we can algebraically express the budget space in the following inequality form :

$$PxX + PyY \leq M, \text{ or } M \geq PxX + PyY$$

The budget space is the entire area enclosed by the budget line AB and the two axes.

Changes in Price and Shift in Budget line:

Now, what happens to the budget line if either the prices of goods change or the income changes.

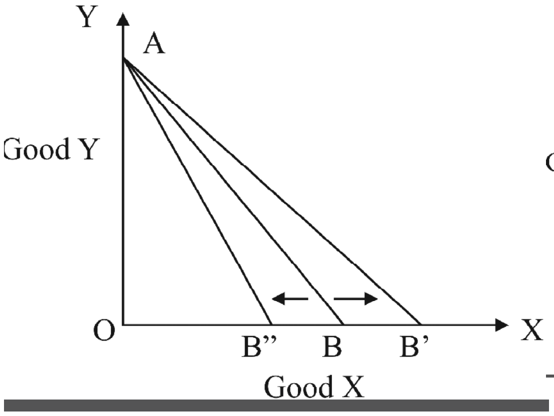


Fig 1.3 Changes on Budget line as a result of change in price of Good X

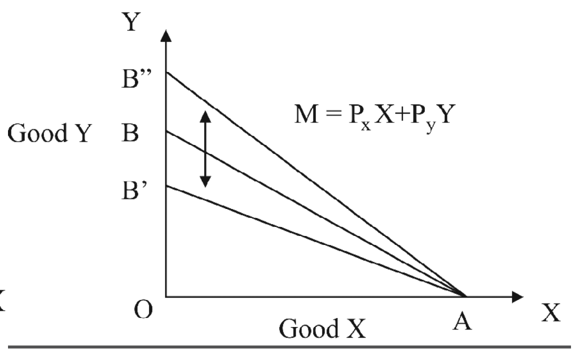


Fig 1.4 Changes on Budget line as a result of change in price of Good X

Let us first take the case of the changes in prices of the goods. This is illustrated in Figure 1.3-1.4. Suppose the budget line in the beginning is AB, given certain prices of goods X and Y and a certain income remaining unchanged. Now, with a lower price of X the consumer will be able to purchase more quantity of X than before with his given income.

Let at the lower price of X, the given income purchases OB' of X which is greater than OB, Since price of Y remain the same, there can be no change in the quantity purchased of good Y with the same given income and as a result there will be no shift in the point A. Thus, with the fall in price of Y remaining constant, the budget line will shift to the right to the new position A'B'.

Now, what will happen to the budget line (initial budget line AB) if price of good X rises, the price of good Y and income remaining unaltered. With higher price of good X, the consumer can purchase smaller quantity of X. Thus, with the rise in price of X the budget line will shift to the left to the new positions A"B".

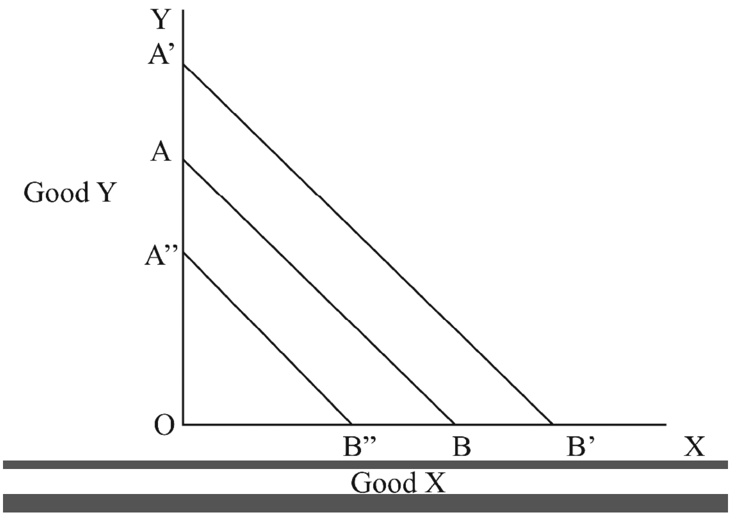


Fig 1.5

Figure 1.4 shows the changes in the budget line when price of good Y falls or rises, with the price of X and income remaining the same. In this the initial budget line is AB. With fall in price of good Y, other things remaining unchanged, the consumer could buy more of Y with the given money income and therefore budget line will shift above to AB'. Similarly, with the rise in price of Y, other things being constant, the budget line will shift below to AB".

Changes in income and shifts in Budget line :

Now, the question is what happens to the budget line if income changes, while the prices of goods A remain the same. The effect of changes in income on the budget line is shown in Fig.1.5. Let AB be the initial budget line, given certain prices of goods and income. If consumer's income increases, the price line shifts upward (say, to A'B') and is parallel to the original budget line AB.

This is because with the increased income, the consumer is able to purchase proportionately larger quantity of good X than before if whole of the income is spent on X, and proportionally greater quantity of good Y than before if whole of the income is spent on Y. On the other hand, if income of the consumer decreased, prices of both goods X and Y remaining unchanged, the budget line shifts downward (say, to A"B") but remains parallel to the original price line AB.

This is because a lower income will purchase a proportionally smaller quantity of good X if the whole of the income is spent on X and proportionately smaller quantity of good Y if the whole of the income is spent on Y.

It is clear from above that the budget line will change if either the prices of goods change or the income of the consumer changes.

Thus, the two determinants of the budget line are :

(a) *The price of goods,* and

(b) *The consumer's income to be spent on the goods.*

1.4. Cardinal Vs. Ordinal Utility

Utility is want satisfying power of a commodity. Consumer derives satisfaction from consumption. Utility is the satisfaction or pleasure that an individual derives from the consumption of a goods or service.

The total amount of such satisfaction is **total utility**. The satisfaction from the last unit is the marginal utility. There are two types of utility. Cardinal utility (utility measured in units) and Ordinal utility (utility revealed through preferences).

The economists of 18th century and 19th century like Alfred Marshall (1842-1924) believed utility could be expressed as an absolute quantity, one, two, three etc., known as Cardinal Utility. And they coined the term UTIL (a unit in which economists imagine that they can measure utility) to refer to the satisfaction derived from consuming a product. Later economists in the late 1930s like V. F. D. Pareto (1848-1923), A. C. Pigou (1877-1959), Russian economist Eugene Slutsky (1880-1948), J.R. Hicks (1904-1989) believed that utility measured on such a scale was neither necessary nor indeed possible.

Such economists were termed 'ordinalists' because they believed utility was subjective satisfaction and could be measured on an ordinal scale, that is, one where only rankings of preference were required, not magnitudes on a numerical scale. The difference between the two approaches can be highlighted by which they view utility.

The **cardinal approach** holds that rational consumers will equalize the utility derived from the marginal unit of cash spent on each item. A rise in the price of eggs thus requires that consumers raise the marginal utility they derived from eggs. Given the assumption of diminishing marginal utility, the only way to effect such an increase is to cut consumption to a point at which eggs are more appreciated than they were and their marginal utility rises.

The **ordinal approach** is only concerned with the relative attractiveness of items. It the ratio of marginal utilities that is important and neither the consumer nor the economist needs to rely on a concept of utilities of some absolute value. In this approach, consumers will ensure that the ratio of prices of items equals the ratio at which the consumer would choose to swap the items with indifference.

Limitations of cardinal utility theory

Limitations of cardinal Utility theory:

(1) There are *no inferior goods* in the cardinal utility theory;

(2) There are no *Giffen goods.* Since Giffen goods are inferior goods , they are automatically excluded when there are no inferior goods. There are no gross substitutes or complements in the model. The change in the price of one good does not affect the demand for any other good;

(3) The marginal utility of money is not necessarily constant in reality. If marginal utility of money is variable, diminishing marginal utility is not sufficient for equilibrium;

(4) The law of diminishing marginal utility rests on a *strong cet.par. clause*;

(5) The marginal utility of money may not decline as income increases;

(6) The strongly additive form of the utility function is not justified;

(7) Utility cannot be measured 'cardinally'.

Replacement of cardinal utility by the ordinal utility theory

To overcome the above limitations, Hicks and Allen (1934) revived and developed the use of generalized utility functions to analyze consumer behavior. Their theory of consumer behavior was able to accommodate cross price effects, some inferior goods, and even Giffen goods.

The Hicks-Allen theory abandoned the cramping dependence on diminishing marginal utility and constancy of marginal utility of money. The former was no longer necessary or sufficient for the consumer's equilibrium. This was now characterized by the convexity of ' indifference curves'.

The cardinal measurement of utility disappeared with the abandonment of constancy of marginal utility of money. It was no longer maintained that the consumer will answer the question: 'How much more satisfaction'. Instead it was asserted that consumer could rank his satisfaction *in an ordering.* Hence it's other name, 'Ordinal utility theory.

Limitations of the Ordinal utility theory:

The chief defect of the ordinal utility theory is that it is based on it is based on psychological attributes which cannot be directly observed. To overcome this limitation, Samuelson developed the theory of revealed preference which is entirely based on observable behavior of a consumer, and some assumptions about his rationality. When confronted with this theory, Hicks admitted the limitations of the ordinal utility approach to consumer demand

Apart from this, the ordinal utility theory has to make numerous assumptions for mathematical and logical convenience, which may not be justified.

1.5 Indifference Curves

Economists in the late 1930s like V. F. D. Pareto (1848-1923), A.C. Pigou (1877-1959), Russian economist Eugen Slutsky (1880-1948), J.R. Hicks (1904-1989) believed that utility measured on the basis of utils of utility was neither necessary nor indeed possible. They were called 'ordinalists' because they believed utility was subjective satisfaction and could be measured on an ordinal scale, that is, one where only rankings of preference were required, not magnitudes on a numerical scale. In this connection we may mention John E. Romer's view : With the 20th century ordinal revolution, the utility function began to be viewed only as a representation of an individual preference order over commodities, rather than an absolute measure of interpersonally comparable welfare.

Utility function

Utility function is generally expressed in the form U = f (x1, x2, x3, x4, x5, …). It shows a consumer's utility as a function of the quantities of goods and services 1,2,3,4,5, … that he or she consumes.

Indifference curve approach

The simple model of consumer behaviour here will enable us to predict how much of a particular commodity — hot dogs, paint, housing — a consumer will buy during a particular period of time.

Economists begin by assuming that the consumer has a fixed amount of money, whole of which is to be spent on the two goods, given constant prices of both the goods.

Preferences are **complete** in the sense that all consumers would be able to decide whether they preferred the first market basket to the second, whether they preferred the second to the first, or whether they are indifferent between them. This certainly seems to be a plausible assumption.

Second, economists assume that the consumer's preferences are **transitive** — that is, if he or she prefers Pepsi to Coke and Coke to Mirinda, then he or she would prefer Pepsi to Mirinda.

Third, economists generally make a **nonsatiation** assumption so that consumers always prefer more of a commodity to less. Suppose that one basket contains 10 computer disks and 2 cups of coffee and a second contains 10 computer disks and 1 cup of coffee. We assume that the first basket, which unambiguously contains more commodities, would be preferred. But there is more to it than that. Implicit in the "more is better" assumption is the notion that we can make the second basket equally desirable in the eyes of the consumer by adding more computer disks even if we hold the coffee allocation fixed at 1 cup. We assume that we can make the consumer indifferent between the two alternatives. These assumptions, like the previous two, seem quite plausible.

The Indifference Curve

The technique of Indifference Curves was invented by F. Y. Edgeworth and redefined by Pareto and Fisher. But it had never been popular and subsequently fell into disuse. It was revived by A.L. Bowley in his *Mathematical Groundwork (1924)*; Bowley did not, however, explore its implications for the measurability of utility. In 1934, J.R. Hicks and R.G.D. Allen showed that indifference curves can be employed to reconstruct the theory of consumer behaviour on the basis of ordinal utility. Johnson and Eugen Slutsky had independently demonstrated the same results as early as 1913 and 1915. The advantage of the Indifference Curve technique is that it forces us by virtue of the concept of 'indifference' itself to pay attention to the interrelationship between goods.

It turns out that we can represent any consumer's tastes or preferences by a set of "indifference curves" if the assumptions hold true. What is an indifference curve? It is a curve, plotted in terms of units of alternative goods, that shows those market baskets (combination of goods) among which the consumer is indifferent.

Assumptions of Indifference Curves

The various assumptions of indifference curve are :

1. **Two commodities :** It is assumed that the consumer has a fixed amount of money, whole of which is to be spent on the two goods, given constant prices of both the goods.
2. **Concept of Non Satiety :** It is assumed that the consumer has not reached the point of saturation. Consumer always prefer more of both commodities, i.e. he always tries to move to a higher indifference curve to get higher and higher satisfaction.
3. **Transitivity :** If x, y and z are any three commodity combinations and if x is indifferent with y and y is indifferent with z, then the consumer is also indifferent between x and z. This condition simply requires that the consumer's tastes possess a simple type of consistency.
4. **Ordinal Utility :** Consumer can rank his preferences on the basis of the satisfaction from each bundle of goods.

5. **Diminishing marginal rate of substitution :** Indifference curve analysis assumes diminishing marginal rate of substitution. Due to this assumption, an indifference curve is convex to the origin.

6. **Rational Consumer :** The consumer is assumed to behave in a rational manner, i.e. he aims to maximize his total satisfaction.

These assumptions permit us to deduce the properties of indifference curves.

Indifference curve refers to the graphical representation of various alternative combinations of bundles of two goods among which the consumer is indifferent. Alternately, indifference curve is a locus of points that show such combinations of two commodities which give the consumer same satisfaction.

If again we assume two products, X and Y, it is possible to possible to produce an indifference schedule which shows different combinations of the two products which yield the same level of satisfaction.

Table 1.1 : An Indifference Schedule

Combination	Units of Product X	Units of Product Y
A	10	30
B	20	16
C	30	9
D	40	5

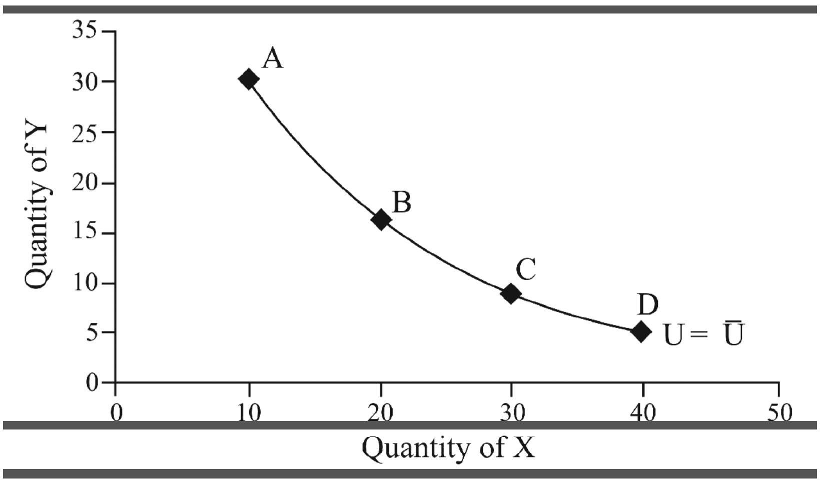


Fig: 1.6: An Indifference curve

Table 1.1 can be represented diagrammatically and the individual obtains the same level of satisfaction whether at point A, B, C or D or, in fact, at any points in between. It is possible to define an indifference curve as a line that joins all the combinations of two products which give the consumer the same level of satisfaction i.e., utility U = –U.

The Indifference Map

So far, we have constructed only a single indifference curve (Fig. 1.6). However, starting at any other point there will be other combinations that will yield equal utility to the consumer. If the points, indicating all of these combinations are connected, they will form another indifference curve. This exercise can be repeated as many times as we wish. The farther any indifference curve as we wish. The farther any indifference curve is from the origin, the higher will be the level of utility given by any of the points on the curve (i.e., $IC_3 > IC_2 > IC_1$). *A set of indifference curves is called an indifference map* (Fig. 1.7).

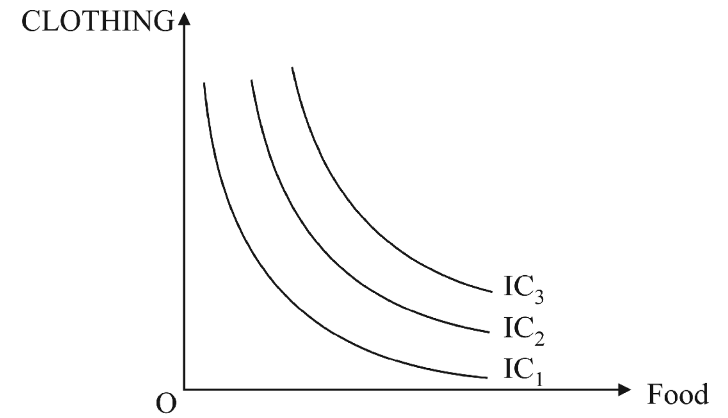


Fig: 1.7: An Indifference map

Properties or Characteristics of Indifference Curve

(a) **An indifference curve is negatively sloped :** The indifference curves must slope downward from left to right. As the consumer increases the consumption of good X, he has to give up certain units of good Y in order to maintain the same level of satisfaction.

(b) **An indifference curve is convex to the origin :** It means that the indifference curve is bent towards the origin. The absolute slope of an indifference curve diminishes toward the right (the curve is flatter at point D in Fig 1.6 than it is at point A) so that the curve is said to be convex to the origin. This property is a direct consequence of the assumption of diminishing marginal rate of substitution, which states that the marginal rate of substitution of X for Y is smaller at D than at A (Fig 1.6)

(c) **Higher and higher indifference curve means higher and higher level of satisfaction :** It is possible to produce an indifference map. There are an infinite number of indifference curves although only three are shown in the Figure 1.7, and a move to an indifference curve further from the origin, say from IC_1 to IC_2 represents an increase in the level of utility or satisfaction. This is because indifference curves further from the origin represent consumption bundles containing more of both products, which means higher utility or satisfaction.

(d) **Two Iso-utility or indifference curves cannot intersect each other:** Indifference curves cannot intersect as shown in Fig. 1.8. Consider the points A_1, A_2 and A_3. Let the consumer derive the satisfaction U1 from the batch of commodities represented by A1 and similarly U_2 and U_3 from A_2 and A_3. The consumer has more of both commodities at A3 than at A1, and therefore $U_3>U_1$. Since A_1 and A_2 are on the same indifference curve, $U_1 = U_2$. The points A_2 and A_3 are also on the same indifference curve, and therefore $U_2 = U_3$. This implies $U = U_3$.

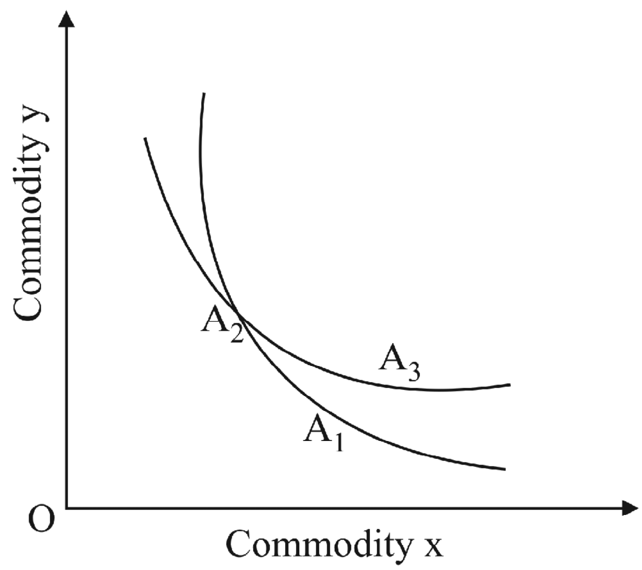


Fig 1.8: Indifference curve cannot intersect

Therefore A_1 and A_3 are one the same indifference curve, contrary to assumption. Hence two indifference curves cannot intersect.

Marginal Rate of Substitution (MRS)

The marginal rate of substitution is defined as the (maximum) number of units of good Y that the consumer would willingly sacrifice in return for an extra unit of good X while still keeping his or her level of satisfaction unchanged. Fig. 1.9 , for example, shows that the consumer would relinquish $(Y_2 - Y_1)$ units of good Y to receive $(X_1 - X_2)$ more units of good X, and this trade would leave him or her no better and no worse off. How do we know? Because both allocations lie on the same indifference curve, the marginal rate of substitution of good X for good Y in this case, then, is $(Y_2 - Y_1) / (X_1 - X_2)$.

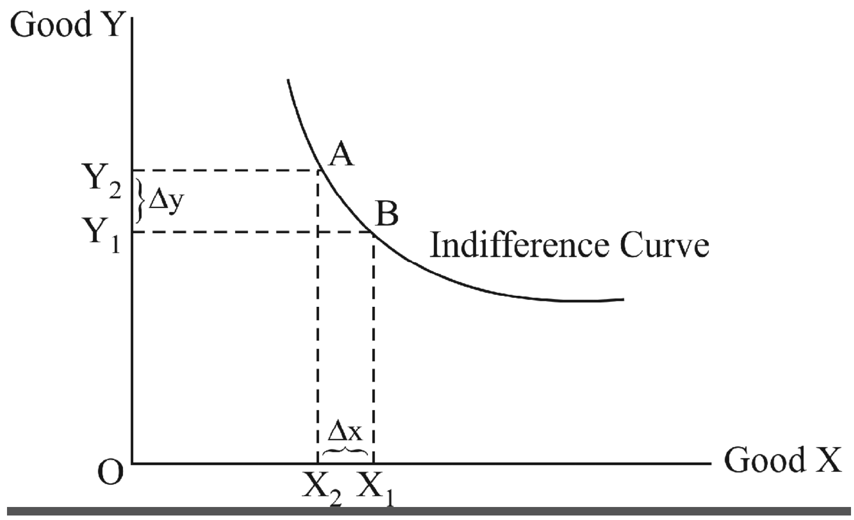


Fig:1.9 Marginal rate of substitution

Exceptional Indifference Curves

Following are the shapes of exceptional indifference curves :

Shape of the indifference curve showing perfect substitutability between good X and Y :

For X and Y to be perfect substitutes, the MRS_{xy} must be constant. That is, no matter what indifference curve we are on and where we are on it, we must give up the same amount of Y to get one additional unit of X. For example, in going from one point A to B on the indifference curve IC_1, the MRS_{xy} is constant. If the indifference curves had throughout a slope of -1 then it would be a case of perfect substitutes [Fig.

1.10(A)]. And in that case, indifference curve would be a negatively sloped straight line. For example, nickel and dimes can be considered as perfect substitutes.

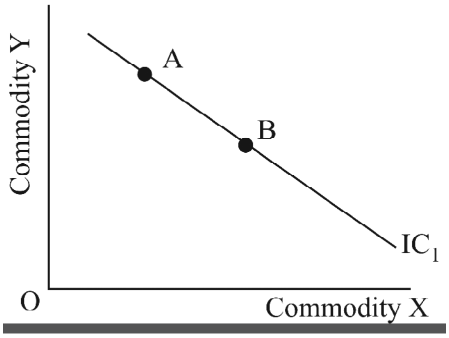


Fig: 1.10(A) : Shape of the indifference curve showing perfect substitutes

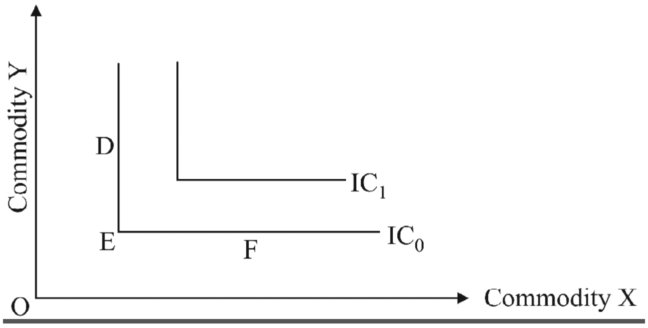


Fig:1.10(B) : Shape of the indifference curve showing perfect complementarity

Shape of the indifference curve showing perfect complementarity between good X and Y

For X and Y to be perfect complements, the MRS_{xy} and the MRS_{yx}, must both be equal to zero. For example, points D, E and F are all on the indifference curve IC_0, yet F involves the same amount of Y but more of X than point E [Fig. 1.10(B)]. Thus, the consumer is saturated with X and so the $MRS_{xy} = 0$. Similarly, point D involves the same amount of X but more of Y than point E, thus the consumer is saturated with Y and so $MRS_{yx} = 0$. For example, left shoe and right shoe can be considered as perfectly complementary goods.

Utility maximisation of a consumer subject to limited income: Consumer Equilibrium

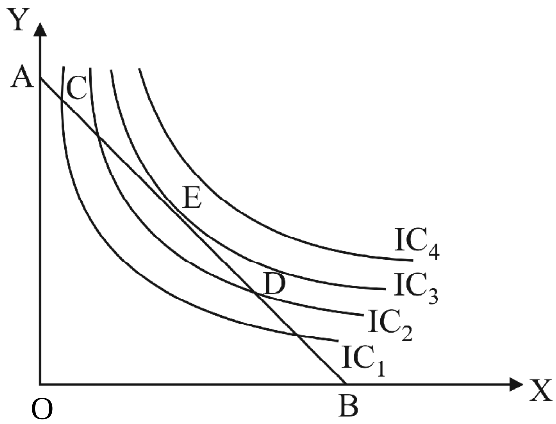


Fig: 1.11 Consumer's equilibrium

Example: If the utility function for two goods $X_1$and X_2 is given by:

$U(X_1, X_2,)=X_1^{\alpha} X_2^{1-\alpha}$, where $0<\alpha<1$

We can calculate the demand functions for X_1 and $X_{2,}$ as functions of P_1, P_2 and money income(M)

[Ans Hints**:** The Lagrangean function is: $V= X_1^{\alpha} X_2^{1-\alpha} + \lambda$ (M— P_1X_1—P_1X_2).

Maximization of utility subject to the budget constraint M— P_1X_1— P_1X_2 yields the demand functions

$X_1= \alpha M/P_1$ and $X_2= (1-\alpha).M/ P_2$]

Two Special cases about consumer's equilibrium resulting corner solution

In the first case (Fig:1.12) the indifference curves are concave rather than convex, i.e., the assumption that the utility function is quasi-cocave is violated and thus shows increasing MRCS (marginal rate of commodity substitution).The first order condition for a maximum is satisfied at the point of tangency (A) between the budget line and an indifference curve, but the second order condition is not. Therefore, this point represents a local utility minimum. And he consumes only one commodity at the optimum. Here,, shown in Fig 1.12, he will buy only quantity of X.

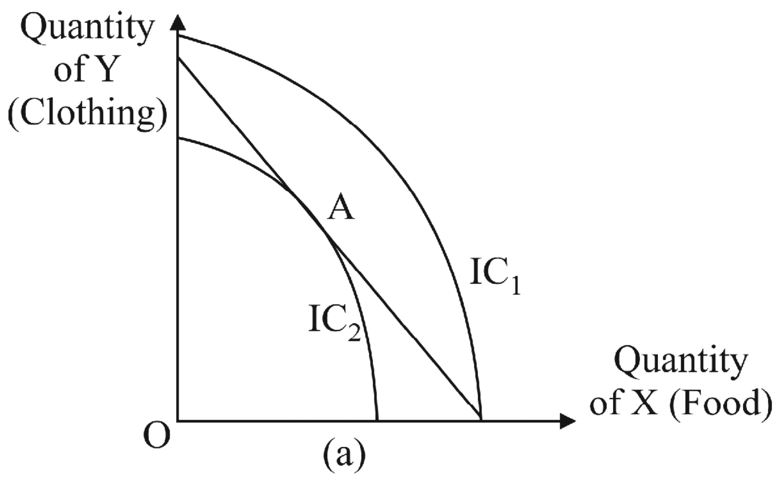


Fig: 1.12 : An exceptional indifference curve

Therefore, this point represents a local utility minimum. And he consumes only one commodity at the optimum. Here, shown in Fig 1.12, he will buy only quantity of X.

In the second case (Fig1.13), the indifference curve s have the appropriate shape but they are everywhere less steep than the budget line. Tangency is not possible. The consumer's optimum position is again given by a corner solution, and he purchases only quantity of Y at the optimum.

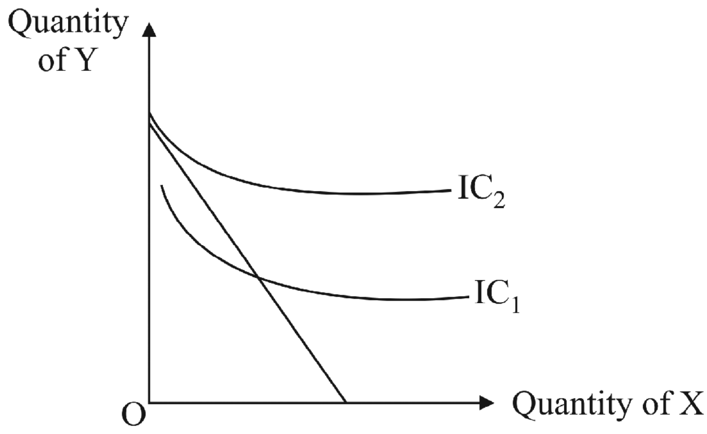


Fig:1.13 An Exceptional Indifference curve

1.6 Revealed Preference Theory

Revealed preference theory is an approach to consumer theory pioneered by Samuelson(1938) in place of cardinal theory or indifference curve methods; an empirical utility theory. It does not require complete information about a consumer's tastes but only knowledge of the combinations of goods actually purchased out of a consumer's total income. In other words, It makes no reference to the concept of utility.The concept of revealed preference states that preferences are essentially defined by what we choose. If Asim chooses ice-cream to salad, then it follows that the he prefers ice-cream to salad.

The theory of revealed preference rests on the following assumptions:

(1) There is *consistency*; that is, if the consumer is observed to prefer basket A to basket B, then this consumer will never prefer B to A;

(2) The individual's tastes do not change over the period considered;

(3) There is *transitivity*; that is, if A is preferred to B and B to C, then A is preferred to C;

(4) Finally, the consumer can be induced to purchase any basket of goods if its price is made sufficiently attractive.

This Samuelsonian version has proved to be both in-tuitively attractive and very powerful in its ability to get results. The basic idea of the theory is very simple. We first define a plausible "principle of rationality" and then show how this principle of rationality allows us to approximate any indifference curve as closely as we like. In this sense a person who obeys the principle of rationality is operating as if maximizing a utility function.

Graphic Approach

The principle of rationality in the theory of revealed preference is as fol-lows: Let us consider two bundles of goods, *A* and *B.* If at some prices and income level, the individual can afford both *A* and B but chooses *A, we* say that *A* has been "revealed preferred" to B.

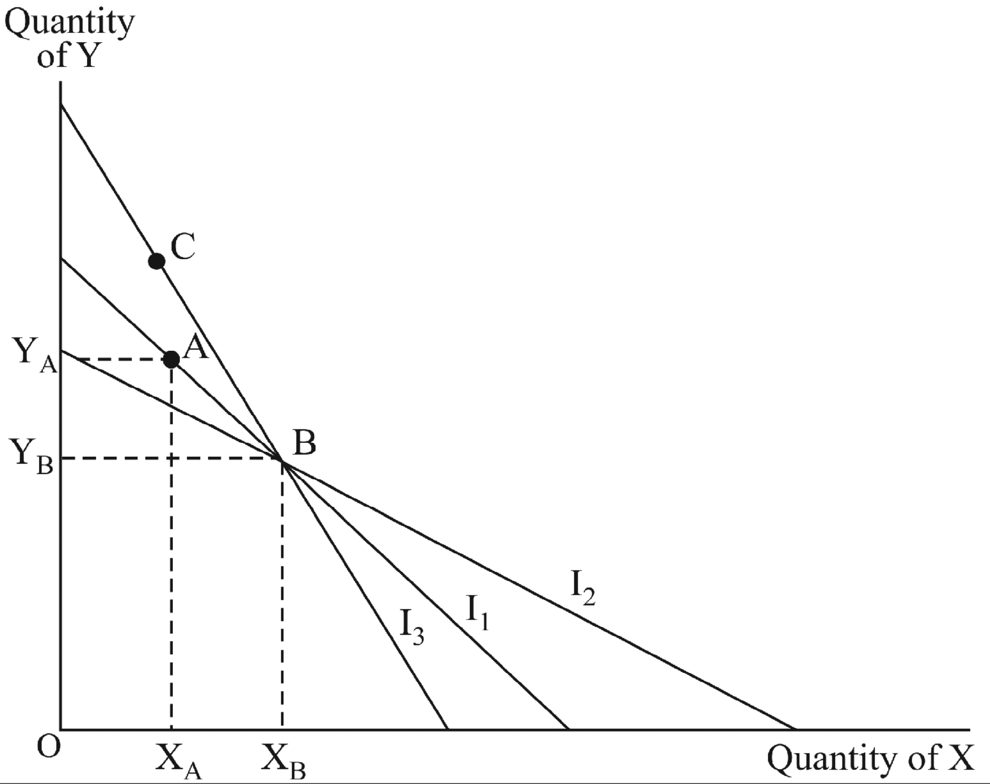


Fig 1.14 : Demonstration of the Revealed Preference Axiom of Rationality

With income I_1 the individual can afford both points *A* and *B*. If *A* is selected, A is revealed preferred to B. It would be irrational for B to be revealed preferred to A in some other price- income configuration

The principle of rationality states that under any different price-income arrangement, B can never be revealed pre-ferred to *A*. If B is in fact chosen at some price-income configuration, it must be because the individual cannot afford *A*. The principle is illustrated in Figure 1.14. Suppose that when the budget constraint is given by I_1, point *A* is chosen, even though B also could have been purchased. *A* then has been revealed preferred to B. If for some other budget constraint, B is in fact chosen, it must be a case such as that represented by $\mathbf{I_2}$—where *A* could not have been bought. If B were chosen when the budget constraint is I_3, this would be a violation of the principle of rationality, since with I_3 both *A* and B can be bought. With budget constraint I_3 it is likely that some point other than either *A* or B, say, C, will be bought.

This principle of rationality can be used to show a variety of results about the consumer behavior. Perhaps the most instructive example would be to use the Revealed preference to show that the substitution effect must be negative.

Axioms of revealed preference

There are two axioms of revealed preference: Strong Axiom and Weak Axiom.

Strong Axiom of Revealed preference:

The *strong axiom of revealed preference* states that if commodity bundle 0 is revealed preferred to bundle 1 , and if bundle 1 is revealed preferred to bundle 2, and if bundle 2 is revealed preferred to bundle 3,…, and if bundle K-1 is revealed preferred to bundle K, then bundle K cannot be revealed preferred to bundle 0 (where K is an arbitrary number of commodity bundles.).

Weak Axiom of Revealed Preference

If commodity bundle **q0** is revealed to be preferred to commodity bundle $\mathbf{q^1}$**,** the latter $\mathbf{q^1}$must never be revealed to be preferred to commodity bundle **q0** in any other price situation**.** The only way that $\mathbf{q^1}$ is revealed to be preferred $\mathbf{q^0}$ at price $\mathbf{p^1}$ if

$$\mathbf{p^1 q0 \le p^1 q^1}$$

Through revealed preference theory, we can deduce that demand functions are homogeneous of degree 0 in all prices and income and the Hicksian cross substitution terms are identical. It is therefore is apparent that that the revealed preference axiom and the existence of of “well-’behaved” utility functions are somehow equivalent conditions. That this is in fact the case was first shown by H. Houthakker in 1950. He showed that a set of indifference curves can always be derived for an individual that obeys the strong axiom of revealed preference. Hence this axiom provides a quite general and believable foundation for utility theory based on simple comparisons among alternative budget constraints.

Example: A consumer is observed to purchase q_1=20, q_2=10 at prices p_1=2, p_2= 6. She is also observed to purchase q_1=18, q_2= 4 at the prices p_1=3, p_2=5. Is her behavior consistent with the axiom of the Theory of Revealed Preference?

[Ans: Let q_1^0 =20 and q_2^0 = 10 represents the set q^0 & p_1^0 = 2 and p_2^0 = 6 represents the set p^0

So the purchase of this set costs to $P_1^0q_1^0+ p_2^0q_2^0 =2\times20+6\times10=100= p^0 q^{0,}$

And let p_1' =3 and p_2' =5 represents by p'

& q_1' =18 and q_2'=4 represents by q'

The purchase of this set costs $p_1' q_1' + p_2' q_2' = 3\times18+5\times4 = 74 = p' q'$

As p^0q^0 =100 & p^0q' =60 , then we have $p^0q'<p^0q^0$. But still she purchases q^0.So q^0 is revealed to be preferred to q’.

Now we have $p' q'$ = 74 and $p'q^0$ =20×3+10×5=110

So we have $p^1q^0 >p'q'$

And the consumer purchases q^0 even when p^0q^0e”p^0q’ and we also know that $p'q^0$ $>p'q'$

So the behavior of the consumer is consistent with the weak axiom of revealed preference.]

Limitations of the Revealed Preference Theory

Revealed preference is observable behavior. This is because, in any price situation, the consumer can be seen to choose some commodity bundle in preference to others which are available. The revealed preference theory has some advantages in explaining consumer behavior. It is imbibed with simplicity. Yet it suffers from two fundamental defects.

First, even when the individual consumers are rational, their aggregate behavior in the market may not satisfy the axioms of revealed preference. The theory can be safely applied only to the individual behavior but not to the aggregate behavior of consumers.

Secondly, the theory cannot be used in welfare economics.

Superiority of revealed preference theory:

The revealed preference theory is distinctly superior to indifference curve approach in predicting a consumer's reaction to single price change which generates income and substitution effect. It is also possible to derive the homogeneity property of the demand function from the axioms of revealed preference.

1.7 Duality approach

The word 'duality' is often used to make a contrast between two related concepts. In microeconomic approach, however, duality refers to connections between quantities and prices which arise as a consequence of the hypotheses of optimization and convexity. Connected to this duality is the relationship between utility and expenditure functions etc. These dual relationships are not naturally a product of the calculus; they are rooted in convex analysis, and in particular, in different ways of describing a convex set.

In short, duality is the relationship between any constrained maximization problem and its related "dual" constrained minimization problem .

1.7.1 Indirect Utility Function and its Properties

Its first discovery was made as early as 1886 by Antonelli in Italy. Later contributions came from Russia, from Hotelling (1932) and Court (1941) in the United States and from Wold (1943-4) in Sweden.

But it was not until the early 1950s and the contributions of Houthakkar (1951-2,1960) that the indirect utility function (a representation of utility as a function of all prices and income) became an integral part of the theory of consumer's behavior.

A consumer's indirect utility function is a function of prices of goods and the consumer's income or budget. The function is typically denoted as $v(p, m)$ where p is a vector of prices for goods, and *m* is a budget presented in the same units as the prices. The indirect utility function takes the value of the maximum utility that can be achieved by spending the budget *m* on the consumption goods with prices *p*. This function is termed "indirect" because consumers generally consider their preferences in terms of what they consume rather than price (as is used in the function).

The indirect utility function is of particular importance in microeconomic theory as it adds value to the continual development of consumer choice theory and applied microeconomic theory. Related to the indirect utility function is the expenditure function, which provides the minimum amount of money or income an individual must spend to achieve some pre-defined level of utility. In microeconomics, a consumer's indirect utility function illustrates both the consumer's preferences and prevailing market conditions and the economic environment.

Indirect Utility Function and UMP (utility maximization problem).

The indirect utility function is closely related to the utility maximization problem (UMP). In microeconomics, the UMP is an optimal decision problem that refers to the problem consumers face with regards to how to spend money in order to maximize utility. The indirect utility function is the value function, or the best possible value of the objective, of the utility maximization problem:

$$v(p, m) = \max u(x) \quad \text{subject to} \quad p \cdot x \leq m$$

Property of the indirect utility function

It is important to note that in the utility maximization problem consumers are assumed to be rational and locally non-satiated with convex preferences that maximize utility. As a result of the function's relationship with the UMP, this assumption applies to the indirect utility function as well.

Another important **property** of the **indirect utility function** is that it is degree-zero homogeneous function, meaning that if prices (*p*) and income (*m*) are both multiplied by the same constant the optimal does not change (it has no impact).

Last, but not least, the indirect utility function is also quasi-convex in prices.

How to get indirect utility function from direct utility function

We can, for example an indirect utility function that corresponds to the direct utility function U=á $\log q_1 + q_2$.

Ans:

Maximization of utility subject to the budget constraint $v_1q_1 + v_2q_2 = 1$ yields the demand functions

$$q_1 = \frac{\alpha v_2}{v_2} \qquad q = \frac{1}{v_2} - \alpha$$

and the indirect utility function

$$U = \alpha \ln\left(\frac{\alpha v_1}{v_1}\right) + \frac{1}{v_2} - \alpha$$

with the derivatives

$$\frac{\partial U}{\partial v_1} = -\frac{\alpha}{v_1} \qquad \frac{\partial U}{\partial v_2} = -\frac{1-\alpha v_2}{v_2^2}$$

1.7.2 Expenditure Function:

Expenditure functions are frequently used to describe the preferences of consumers.

Individual's dual expenditure minimization problem is to choose $X_1, X_2,\ldots, X_n$ so as to minimize:

total expenditure=E= $P_1X_1+P_1X_2 +\ldots P_nX_n$

subject to the constraint

utility = $U_2= U(X_1, X_2,\ldots, X_n)$

The optimal amounts of $X_1, X_2,\ldots, X_n$ chosen in this problem will depend on the prices of the various goods($P_1,P_2,\ldots,P_n$) and the required utility level U_2. If any of the prices were to change or the individual had a different utility "target," another commodity bundle is optimal. This dependence can be said by an *expenditure function* which shows the minimal expenditures necessary to achieve a given utility level for a particular set of prices. That is,

Minimal expenditure = E $(P_1,P_2,\ldots,P_n, U)$

1.8 Consumer Surplus

Although it is generally accepted that J. Dupuit (1804-1866) was the originator of the concept of consumer surplus, it is largely attributed to Alfred Marshall who expressed his opinion in his famous book *Principles of Economics*. It is a satisfaction that consumers obtain from a good over and above the price paid. This is the difference between the maximum demand price that you would be willing to pay and the price that you actually pay. For most consumers, under most circumstances, the demand price is greater than the price paid. Even competitive markets overflowing with efficiency generate an ample amount of consumer surplus.

The demand curve can be viewed as a willingness-to-pay curve. It shows the value that consumers place on extra units of the good. **Consumer surplus** is the difference between the amount that consumers actually pay and the amount that they would have been willing to pay. On a graph, consumer surplus can be shown as the area under the demand curve and above the prevailing market price.

The consumer surplus may be found from the demand curve. Assume that the straight line AB denotes the consumer's demand for X and the market price is P. At this price consumer buys q units of X and pays an amount q*P for it. However he would be willing to pay P1 for q1, P2 for q2, P3 for q3 and so on (Figure 1.15).

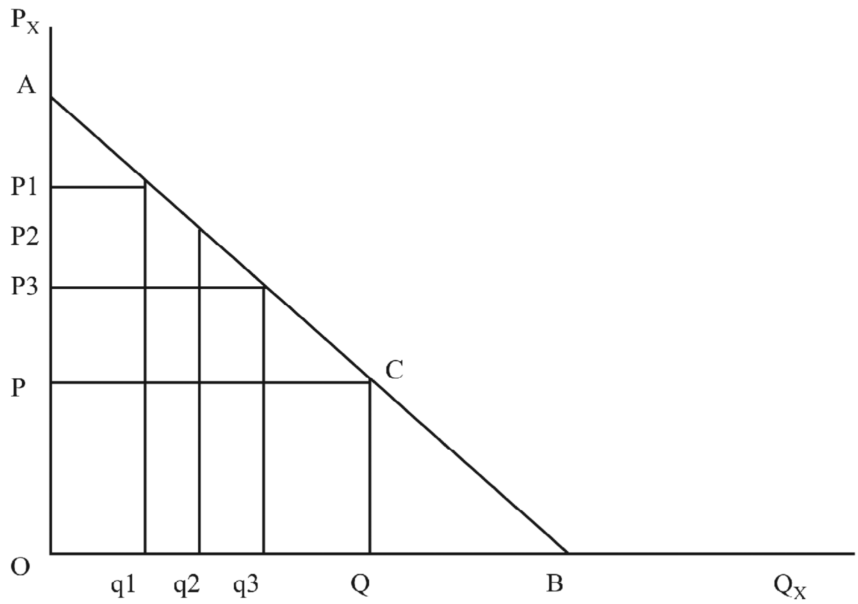


Fig: 1.15 Consumer Surplus

It is evident from the figure that the consumer is willing to pay p1 for q1, but he is actually may the market price P which is less than P1. So PP1 is the consumer's surplus for q1. For each unit upto q, the consumer will gain a suplus. So, the total consumer's surplus may be denoted by the area of the triangle APC. In other words, to purchase q units of output, the consumer is willing to pay the area AOqC, but he is actually paying POqC (Figure 1.16).

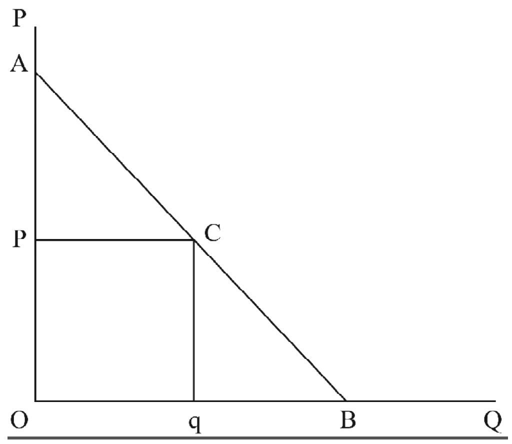


Fig. 1.16: Marshallian consumer surplus

So, the consumer's surplus = AOqC – POqC = APC.

1.8.1 Equivalent and compensating variation

Hicks attempted to rehabilitate the Marshallian concept of consumer's surplus with a new one of his own, the *compensating variation* and *equivalent variation*.

Hick's ' four consumer's surpluses' refer to the compensating and equivalent variation — which measure changes in indirect utility resulting from the changes in prices or income— and to the analogous measure of changes in direct utility resulting from changes in quantities consumed. If we denote by **' x^1R^2** ' the relation 'the bundle x^1 is preferred or indifferent to the bundle x^2', and if the individual's demand function is denoted by **x=h(p,Y)** where p is the vector of prices and Y the individuals income, then the indirect preference relation R* is defined by **' (p^1,Y^1) $R^*(p^2, Y^2)$** if and only if **h(p1,Y) R h(p^2, Y^2)'.** Now from Hurwicz and Uzawa, we get *income compensation function*:

$$\mu^*(p;\, p^0, Y^0) = \min\left\{Y > 01(p,Y)R^*(p^0, Y^0)\right\},$$

where the prices p^0 and income Y^0 define an arbitrary 'base budget', we may define the *price-compensating variation* starting from (p^0, Y^0) and ending at (p, Y) by

$$C^*(p, Y, p^0, Y^0) = Y - m^* (p; p^0 Y^0)$$

and the corresponding *price-equivalent variation* as

$$E^*(p, Y; p^0, Y^0) = \mu^*\left(p^0; p, Y\right) - Y^0$$

These are the 'dual' concepts; the primal ones may be defined analogously if, following Hicks, we decompose the commodity bundle into x=(z,y), where z = (x_1,…, x_{n-1}) and y = x_n, where commodity n is the numeraire. Then the *numeraire*-compensation function is

$$\mu^*(z; z^0, y^0) = \min\left\{Y > 01(z, y)R(z^0, y^0)\right\},$$

a function which is unfortunately not always well defined, since indifference surfaces may touch the axes. The corresponding *quantity-compensating variation* is, of course,

$$C\left(z, y; z^0, y^0\right) = y - \mu\left(z; z^0, y^0\right)$$

and the *quantity-equivalent variation* is

$$E\left(z, y; z^0, y^0\right) = \mu\left(z^0; z, y\right) - y^0.$$

Hicks developed elegant relationships among these 'consumer's surpluses', but the last two have not caught on the literature since μ as mentioned above is not a satisfactory index of indifference in general. As opposed to Marshall's concept of consumer's surplus, Hicks two concepts of Price-compensating variation and price-equivalent variation are exact measure of welfare change between two situations.

Example:

Suppose a consumer has the utility function

U lx,y) = $x^{0.4} y^{0.6}$

We know her demand functions are

$$x = \frac{21}{5P_x} \text{ and } y = \frac{31}{5P_y} \tag{A}$$

and the indirect utility function is

$$V = \frac{2^{0.4}3^{0.6}}{5}\frac{1}{p_x^{0.4}p_y^{0.6}} \tag{B}$$

Now suppose initially our consumer's income is Rs. 100 and the prices she faces are p_x^0 = Rs. 2 and

= Rs. 3 From (A) and (B), we find

x= 20, y_0=20, u_0= 20

Finally suppose p_x falls to Rs.1. Then

x= 40, y_0=20, u_0= 26.39

Our task is to find the CV and EV of this price change.

To find CV. we need to find E(p_1, u_0). Suppose this needed expenditure is the income level I'. We can use (B), using u_0 and p_1, to find

$$20 = \frac{2^{0.4}3^{0.6}}{5}\frac{1'}{1^{0.4}3^{0.6}}$$

which can be solved to yield :

I´ = 75.79

Then, using (A), we find

CV = 75.79 – 100 = – 24.21

Note that welfare change = ΔW_{CV} = 24.21.

To find the EV, we need to find E(p_o, u_1). Let this be I''. Again, using (B), u_1 and the prices p_o we can find:

$$26.39 = \frac{2^{0.4}3^{0.6}}{5}\frac{1''}{2^{0.4}3^{0.6}}$$

which gives us

$$1'' = 131.95$$

Thus, from (A), we find

EV= 131.95 -100 = 31.95

ΔW_{EV} = 31.95

1.9 Conclusion

In what follows is that in this unit we have discussed all about budget line, indifference curve and consumers' equilibrium through the aid of these two apparatus. Second, we have discussed about Samuelson's formulation of revealed preference theory which shows how utility theory can be built up from rather simple comparisons among alternative budget constraints and associated consumer choices. Thereafter we proceed to analyze the consumer's surplus –both Marshallian and Hicksian. along with duality approach.

1.10 Summary

Budget line: a a set of combinations of two commodities that can be purchased if whole of the given income is spent on them and its slope is equal to the negative inverse of the price ratio of the commodities.

Consumer surplus: Consumers' gain from their purchases. The most commonly used such measure is one devised by Alfred Marshall, who regards consumer surplus as the difference between the most the consumers would have willing to pay and what they actually paid. Referring to a diagram with quantity on the horizontal axis and price on the vertical axis, we can calculate Marshallian consumer .surplus as the area beneath demand curve and above the price line.

Marshallian one is very simple and gives benefits to the consumer. If there are "wealth effect" the consumer's willingness to pay differs. There, is thus, not just one measure of consumer surplus, but many. Most prominent are the *equivalent variation* (the additional money needed to make the consumer just as well-off as the price changes) and the *compensating variation* (the money that could be taken away after the price change to leave the consumer as well off as before. The consumer surplus change is bracketed between the equivalent and compensating variation.

Duality: Relationship between any constrained maximization problem and its related "dual" constrained minimization problem.

Indifference curve: In the 1930s most economists became increasingly uncomfortable with the idea of measurement and interpersonal or intergroup comparisons of utility. In 1934, in a famous article entilted' A reconsideration of the Theory of Value', J.R. Hicks and R.G.D. allen used the technique of indifference curves originated by F.Y. Edge worth and developed by Walrus's successor, Vilfredo Pareto, in presenting a theory of consumer behaviour involving only ordinal comparisons of satisfaction.

Indifference map : A set of indifference curves is called an indifference map.

Indirect utility function: A representation of utility as a function of all prices and income.

Revealed preference theory is an approach to consumer theory pioneered by Samuelson in place of cardinal theory or indifference curve methods; an empirical utility theory.According to revealed preference theory, a consumer's preference can be inferred from a sufficient number of observed choices or purchases in the market place, without any need to inquire directly into the individual's preferences.

The Compensating Variation in Income (CV): change in income needed after an economic change has taken place that would be just sufficient to restore the consumer to the original level of utility. Hicks two concepts of Price-compensating variation and price-equivalent variation are exact measure of welfare change between two situations.

The Equivalent Variation in Income *(EV):* the change in income needed before an economic change has taken place that would be just sufficient to bring the consumer to the utility level she would enjoy if the economic change took place.

1.11 Exercises

A. Small-answer Type Questions

1. What is budget line? What are the properties of budget line?

2. What are the limitations of ordinal utility approach?
3. What is revealed preference hypothesis?
4. What is Marshallian consumer surplus ? How can we calculate it?
5. How does Hicks differ from Marshall in theorizing the concept of consumer's surplus?
6. What are the assumptions of the revealed preference theory?
7. What is strong axiom and Weak axiom in the theory of revealed preference?
8. What is duality in consumer's theory?
9. What is indirect utility function?
10. For what purposes the expenditure functions are used?
11. What is compensating variation and Equivalent variation?

B. Medium Answer–type Questions

1. Why has there been a replacement of cardinal utility theory by ordinal one?
2. What are the assumptions of indifference curve?
3. What is indifference curve? What are its properties?
4. What are the shapes of the indifference curve if two goods are (i) complementary goods

   (ii) two goods are substitutes?
5. Discuss two special cases about consumer's equilibrium resulting corner solutions?
6. Suppose the utility function for two goods $X_1$and X_2 is given by:

   $U(X_1, X_2,)=X_1^{\alpha} X_2^{1-\alpha}$, where $0< \alpha <1$

   calculate the demand functions for X_1 and $X_{2,}$ as functions of P_1, P_2 and money income(M)
7. What is indirect utility function? State its properties.

8. A consumer has a utility function of the form $U(x_1,x_{2)}=- 1/x_1- 1/x_2)$

   (a) Compute the Marshallian demand functions

   (b) Show that indirect utility function is $-\left[\sqrt{p_1}+\sqrt{p_2}\right]^2 \Big/ \frac{1}{M}$

9. Construct an indirect utility function that corresponds to the direct utility function $U = \alpha \log q_1 + q_2$.

10. A consumer is observed to purchase $q_1=20$, $q_2=10$ at prices $p_1=2$, $p_2=6$. She is also observed to purchase $q_1=18$, $q_2=4$ at the prices $p_1=3$, $p_2=5$. Is her behavior consistent with the axiom of the Theory of Revealed Preference?

11. Hicks two concepts of Price-compensating variation and price-equivalent variation are exact measure of welfare change between two situations. Do you agree? Give reasons for your answer.

C. Long-answer Type Questions

1. What are the assumptions of indifference curve. What are the properties of indifference curve? Show that two indifference curves never intersect?

2. Discuss the revealed preference theory with diagram with the mentioning of strong and weak axiom? What are the limitations of this theory?

3. What is duality approach in consumer theory? In this connection define indirect utility function and state its properties of it.

4. Show how Hicks rehabilitates the Marshallian concept of consumer's surplus with a new one of his own, the *compensating variation* and *equivalent variation*.

1.12 References

Henderson, A. (1941). Consumer's surplus and the compensating variation, *Review of Economic Studies 9, February:117-21*

Hothakkar, H.S. (1950)'Revealed preference and the utility function', *Economica*, New Series 27:159-74.

Salvatore , D. (1983).*Theory and Problems of Microeconomic Theory,* McGraw Hill, Singapore.

Samuelson,P.A.(1938) 'A note on the pure theory consumer's behaviour', *Economica,* New Series 5:61-71,353-4.

Samuelson,P.A.(1948) 'Consumption Theory in terms of revealed preference', *Economica*, New Series 15:243-53.

Varian, H.R.(1978): *Microeconomic Analysis.* New York: W.w. Norton,Inc., Chapter 3

Unit 2 ❑ Theories of Production, Cost and Profit Maximisation

Structure

2.1 Objectives

After going through this unit you will be able

- To learn budget line and its properties, cardinal vs ordinal utility;
- Indifference curve and its properties and consumer's optimal bundle; and some exceptional indifference curves;
- To have an idea about Samuelson's formulation of revealed preference theory;
- To get a knowledge about duality approach and its associated indirect utility function, its properties and expenditure function; and
- Learn about Marshallian consumer surplus and its Hicksian attempt to rehabilitate Marshallian consumer surplus.

2.2 Introduction

A firm is a technical unit in which commodities are produced. Its entrepreneur (owner and manager) decides how much of and how one or more commodities are produced, gains the profit or bears the loss which results from his decision. An entrepreneur transforms inputs into outputs, subject to the technical rules specified by his production function. The difference between his revenue from the sales of outputs and cost of his inputs is his profit, if positive, or his loss ,if negative.

The entrepreneur's production function gives mathematical expression to the relationship between the quantities of inputs he employs and the quantities of outputs he produces. It is in this context different forms of production functions are laid down. Besides derivation of isoquants and its properties and finding the optimal employment of inputs by isoquant and iso-cost lines are written. All the above is discussed in Section A.2.3 and A.2.4.

In Section B.2.1, an explicit discussion on cost functions is given with special emphasis on learning curve and application of duality in cost function.

In Section C.2.1 the basic assumptions and characteristics of a perfectly competitive market as well as the basic concepts of monopoly are outlined with special emphasis on the concept peak-load pricing.

2.3 Production Function

Production function is a schedule (or table or mathematical equation) showing the maximum amount of output that can be produced from any specified set of inputs given the existing of technology or state of the art. A production function is an expression of quantitative relation between change in inputs and the resulting change in output. It is expressed as

$$Q = f(i_1, i_2 \ldots\ldots i_n)$$

Where Q is output of a specified good and i_1, i_2 ....in are the inputs usable in producing this good. To simplify let us assume that there are only two inputs, labour (L) and capital (K), required to produce a good. The production function then takes the form :

$$Q = f(L,K)$$

Short Run Production Function and Long Run Production Function

The **short run** is a period of time in which at least one of the factors is fixed. The fixed factor is generally taken as capital because it is not possible to construct a new factory or purchase new machinery overnight. It is however, possible to employ extra units of labour in the short run. Hence, the labour is called the variable factor.

The short-run production function can be expressed as

$$Q = f(L,K)$$

For example, a short run production function may be

$$Q = bL$$

The long run is a period of time in which all factors of production can be varied. The **long run** production may be written in the form

$$Q = f(L, K)$$

In microeconomics, conventionally, we study two aspects of relation between inputs and output. One aspect relates to short run: in what manner the change takes place in output of a good, if only one of the inputs required in producing that good is increased, i.e. other inputs kept unchanged? The manner of change in output is summed up in the law of variable proportions which you have already studied.

The second aspect relates to long run : in what manner the output of a good changes, if all the inputs required in producing that good are increased simultaneously and in the same proportion. This aspect is technically termed as returns to scale. The word 'return' refers to the change in physical output. The word 'scale' refers to the scale of operation expressed in terms of quantum of inputs employed.

2.3.1 Various Types of Production Functions

There are various types of production functions, namely linear production function (also known as first degree polynomial function), quadratic production function (also known as second degree polynomial function), square-root production function(that represents a compromise between the Cobb-Douglas and quadratic function), a non-linear production function for fertilizer-crop yield relationship as Mitscherlink-Spillm production function,Cobb-Douglas production function (also called power production function). The Cobb-Douglas production function has been generalized in a number of ways like (i)Transcendental production function; (ii) Zelliner-Revankar production function; (iii)Nerlove-Ringstad production function, and (iv) CES or ACMS production function

But we will focus here only on Leontief production function, homogeneous production function, and its properties, Cobb-Douglas production function and ACMS (or CES production function) with their properties

2.3.2 Leontief Production Function

The simplest among all production functions that are widely used is the *Leontief,* named for the Nobel laureate Wassily Leontief, who devised it. For the two-input case, it is given by

$$Q = \min(aK, bL)$$

So, Q is equal to either *aK* or *bL,* whichever of the two is smaller. Suppose, for example, that $a = 2$, $b = 3$, $K = 4$, and $L = 3$. Then, $Q = \min(2 \times 4, 3 \times 3) = \min(8, 9) = 8$. The isoquant map for $Q = \min(2K, 3L)$ is shown in Figure 2.1.

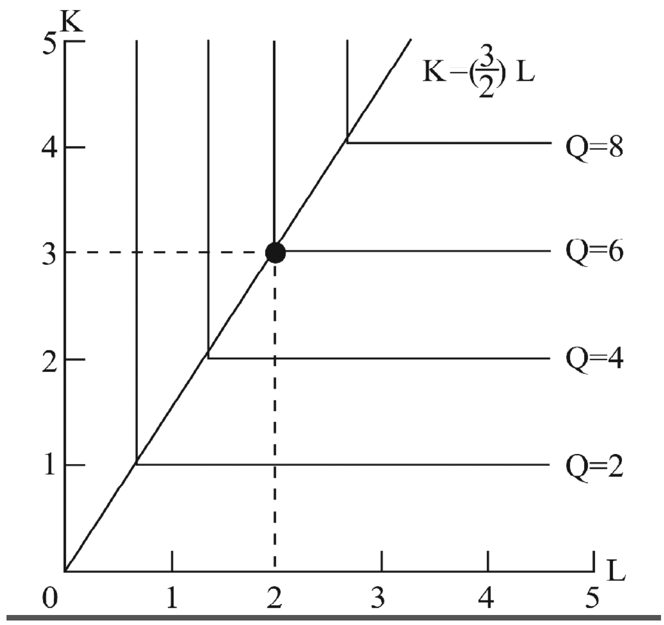


Fig. 2.1 : Isoquant Map for the Leontief Production Function Q = min (2K, 3L)

2.3.3 General Concept of Homogeneous Production Function and its Properties

A production function is homogeneous of degree k if

$$f(tx,ty)= t^k f(x,y)$$

where k is a constant and t is any positive real number. If both inputs (x and y) are increased by the factor t, output is increased by the factor t^k. Returns to scale is increasing if k>1, constant if k=1, and decreasing if 0<k<1.

Properties of Linear Homogeneous Production function:

In the discussion of production function, wide use is made of homogeneous functions of the first degree. They are often referred to as *linearly homogenous* functions,the adverb "linearly" modifying the adjective "homogeneous".

Property 1: Given the linearly homogenous production function Y = f(K, L), the average physical product of labor (APP_L) and of capital (APP_K) can be expressed as functions of the capital labour ratio, K=K/L functions

Property 2: Given the linearly homogenous production function Y = f(K, L), the marginal physical product of labor (MPP_L) and of capital (MPP_K) can be expressed as functions of k alone.

Property 3: If Y= f(K,L) is linearly homogenous, then K. MP_K + L. MP_L=Y. This is Euler's theorem.

2.3.4 Cobb-Douglas Production Function

The Cobb-Douglas (CD) function is perhaps the most ubiquitous form in economics, owing to its popularity and to the fact that it possesses the minimal properties that economists consider desirable. It appeared early by 1916, notably in the theory of distribution where it was used to prove the adding-up theorem of factor shares when the production elasticities sum to unity. It has been applied econometrically countless times, still surprising people that it can explain the data so well. It forces itself into relatively new areas such as frontier production function. And it has been used both as a utility and production function in analysis of growth, development, macroeconomics, public finance, labour and just about any other applied area in economics. Yet it possesses restrictive properties and perhaps for that reason, it has often regarded as a child's toy in the world of real economics. But for others, the Cobb-Douglas is at least a venerable form.

Though it is restrictive and sometimes regarded as an economic toy, the CD form is remarkably robust in a vast majority of applications.

The generalized version of Cobb-Douglas production function is given by

$$Q = AK^{\alpha}L^{\beta}$$

where A is a positive constant and α and β are two positive fractions.

Properties of Cobb-Douglas production function:

Some of the major features of this function are (1) It is homogeneous of degree $(\alpha + \beta)$: (2) in the special case of $(\alpha + \beta) = 1$, it is linearly homogeneous; (3) its isoquants are negatively sloped throughout and strictly convex for positive values of K and L.

Its homogeneity is easily seen from the fact that, by changing K and L to jK and jL respectively, the output will be changed to

$$A(jK)^{\alpha}\ (jL)^{\beta} = j^{\alpha+\beta}\ (AK^{\alpha}L^{\beta}) = J^{\alpha+\beta}\ Q$$

That is, the function is homogeneous of degree (a + b). In case a + b = 1 , there will be constant returns to scale, because the function will be linearly homogeneous.

Implications:

When technical change is allowed to proceed in a Cobb-Douglas world, it is a fact that Hicks—,Solow—and Harrod –neutral technical change are equivalent, thus blurring the distinction. Another Implication of the unit substitution elasticity of the linear homogeneous Cobb-Douglas form is that, used in growth models, it generates the existence and stability of equilibrium growth.

Cobb-Douglas as Representative of Homogenous Production Function

One of the most widely used homogeneous production function is the Cobb-Douglas Function.

The generalized version of Cobb-Douglas production function is given by

$$Q = AK^{\alpha}L^{\beta}$$

where A is a positive constant and α and β are two positive fractions.

By changing K and L to tK and tL respectively, the output will be changed to

$$A(tK)^{\alpha}\ (tL)^{\beta} = t^{\alpha+\beta}\ (AK^{\alpha}L^{\beta}) = t^{\alpha+\beta}\ Q$$

That is, the function is homogeneous of degree $(\alpha + \beta)$. In case $\alpha + \beta = 1$, there will be constant returns to scale, because the function will be linearly homogeneous.

2.3.5 CES Production Function

The limitations of the CD function approach were made strikingly apparent by Arrow, Chenery, Minhas and Solow (1961), henceforth ACMS. The Constant Elasticity of Substitution (CES) Production Function introduced by ACMS adds flexibility to the traditional approach by treating the elasticity of substitution as an unknown parameter. However, the CES production function retains the assumptions of additivity and homogeneity and imposes very stringent limitation on the pattern of substitution. This production function, including its special case the CD form, is perhaps the most frequently employed function in modern economic analysis.

The simplest form of CES production function utilized in production theory is the constant returns to scale type.

$$Y = A\left[\alpha K^{\rho} + (1-\alpha)\, L^{-\rho}\right]^{-1/\rho} \tag{1}$$

where Y = output, K = capital, L= labour ,and the parameter, A reflects the efficiency parameter playing the same role as the A in Cobb-Douglas production function, α is the distribution parameter dealing with the relative factor shares of production function. and ρ is the substitution parameter.

$$A \geq 0,\ 0 \leq \alpha \leq 1 \text{ and } \rho \leq -1.$$

As is implied by its name, the elasticity of factor substitution between capital and labour for production function (1) is expressed as some constant value.

For any neoclassical production function Y=f(K,L), the elasticity of factor substitution between capital and labour is defined as the proportionate change in the K/L ratio (k) relative to the proportionate change in the marginal rate of factor substitution r = marginal product of labour/ marginal product of capital along a given isoquant. That is,

$$\sigma = \frac{diogk}{diogr} \tag{2}$$

where σ represents the elasticity of substitution.

Applying definition (2)to production function (1) we obtain

$$\sigma = \frac{1}{1+\rho} \text{ or } \rho = \frac{1-\sigma}{\sigma} \tag{3}$$

Consequently, it is easy to see why ρ is often referred to as the 'substitution parameter.' The α parameter in the production function (1) is the 'distribution parameter.'

Properties of CES or (ACMS) production function:

(1) This production function is homogeneous of degree one.

Proof:

$$Y = A\left[\alpha\, K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-1/\rho}$$

Changing K to tK and changing L to tL, we get

$$A\left[\alpha\, (tK)^{-\rho} + (1-\alpha)(tL)^{-\rho}\right]^{-1/\rho} = t^1 A\left[\alpha\, K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-1/\rho}$$

Thus it is homogeneous of degree 1.

(2) Average product (AP) and marginal product (MP) are homogeneous of degree zero in the variables K and L

Marginal product of the inputs are:

$$\partial Y/\partial K = \frac{\alpha}{A^{\rho}} \cdot \left(\frac{Y}{K}\right)^{1+\rho}; \text{ and } \quad L = \frac{(1-\alpha)}{A^{\rho}} \cdot \left(\frac{Y}{L}\right)^{1+\rho}$$

(3) The MRTS is decreasing and isoquants convex for ρ > 0. This also establishes the fact that a CES production function is regularly strictly quasi-concave for the domain K, L >0

(4) The iso-quants generated by the CES production function are always negatively sloped and strictly convex for positive values of K and L.

(5) The elasticity of substitution = $\sigma = 1/1+\rho$

Proof:

$$Y = A\left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-\frac{1}{\rho}}$$

or, $$Y^{-\rho} = A^{-\rho}\left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]$$

that is,

$$\frac{Y-\rho}{A-\rho} = \left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]$$

Now, $$MP_K = \frac{\delta Y}{\delta k} = -\frac{1}{\rho} A\left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-\frac{1}{\rho}-1}(-\rho)\alpha k^{-\rho-1}$$

$$= \frac{A\left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-\frac{1}{\rho}}}{\alpha k^{-\rho} + (1-\alpha)L^{-\rho}} \cdot \alpha \frac{1}{k^{\rho+1}}$$

$$= \frac{Y}{\frac{Y^{-\rho}}{A^{-\rho}}} \cdot \frac{\alpha}{k^{\rho+1}} = \frac{Y}{1} \cdot \frac{A^{-\rho}}{Y^{-\rho}} \cdot \frac{\alpha}{k^{\rho+1}}$$

$$= \frac{\alpha}{A^{\rho}} \cdot \left(\frac{Y}{k}\right)^{1+\rho}$$

Similarly, $$MP_L = \frac{\delta Y}{\delta L} = \frac{(1-\alpha)}{A^{\rho}} \cdot \left(\frac{Y}{L}\right)^{1+\rho}$$

The marginal rate of technical substitution(MRTS) shows the rate at which labour can be substituted for capital while holding the output constant along an isoquant. This implies

$$\text{MRTS (L for K)} = \frac{\delta K}{\delta L} = \frac{\delta K}{\delta Y} \cdot \frac{\delta Y}{\delta L} = \frac{\delta Y / \delta L}{\delta Y / \delta K}$$

$$= \frac{\frac{(1-\alpha)}{A^{\rho}}\left(Y/L\right)^{1+\alpha}}{\frac{\alpha}{A^{\rho}}\left(Y/K\right)^{1+\rho}}$$

$$= \frac{(1-\alpha)}{\alpha} \cdot \frac{Y^{1+\rho}}{L^{1+\rho}} \cdot \frac{K^{1+\rho}}{Y^{1+\rho}}$$

$$\frac{\delta K}{\delta L} = \frac{(1-\alpha)}{\alpha}\left(\frac{K}{L}\right)^{1+\rho}$$

Diff. w.r.t $\frac{K}{L}$

$$\frac{\partial\left(\delta K/\delta L\right)}{\partial\left(K/L\right)} = (1+\rho)\cdot\left(\frac{1-\alpha}{\alpha}\right)\cdot\left(\frac{K}{L}\right)^{\rho}$$

For the production function Y= f(K,L), the elasticity of substitution (σ) measures the proportionate change K/L relative to the proportionate change in the MRTS along an isoquant, that is,

Now, $$\sigma = \frac{\frac{\partial\left(\frac{K}{L}\right)}{\frac{K}{L}}}{\frac{\partial\left(\frac{\delta K}{\delta L}\right)}{\frac{\delta K}{\delta L}}} = \frac{\partial\left(K/L\right)}{\partial\left(\delta K/\delta L\right)} \cdot \frac{\delta K/\delta L}{K/L}$$

$$= \frac{1}{(1+\rho)\,\frac{1-\alpha}{\alpha}\left(\frac{K}{L}\right)^{\rho}} \cdot \frac{\frac{1-\alpha}{\alpha}\left(\frac{K}{L}\right)^{1+\rho}}{\frac{K}{L}}$$

$$= \frac{1}{1+\rho} \cdot \frac{\frac{1-\alpha}{\alpha}\left(\frac{K}{L}\right)^{1+\rho}}{\frac{1-\alpha}{\alpha}\left(\frac{K}{L}\right)^{\rho+1}}$$

$$= \frac{1}{1+\rho}.$$

(6) The CES production function is the most well known of the forms that yield the Cobb-Douglas as a special case by using the L/Hopital's rule when the elasticity of substitution goes to infinity. In the extreme case where ρ tends to 0 or σ =1, the CES function (1) converges to the Cobb-Douglas form: $Y=AK^{\alpha}L^{1-\alpha}$ **…(4). In the following expression we take θ instead of ρ**

Proof: We can express the CES production function as follows:

$$Y = \gamma\left[\delta K^{-\theta} + (1-\delta)L^{-\theta}\right]^{-\frac{1}{\theta}}$$

or, $$\lim_{\theta\to 0} Y = \lim_{\theta\to 0} \gamma\left[\delta K^{-\theta} + (1-\delta)L^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$= \frac{\gamma}{\left[\frac{\delta}{K^{\theta}} + \frac{1-\delta}{L^{\theta}}\right]^{\frac{1}{\theta}}}$$

$$= \frac{\gamma}{\left[\frac{\delta L^{\theta} + K^{\theta}(1-\delta)}{K^{\theta}L^{\theta}}\right]^{\frac{1}{\theta}}}$$

$$= \frac{\gamma}{\frac{\left[\delta L^{\theta} + K^{\theta}(1-\delta)\right]^{\frac{1}{\theta}}}{\left(K^{\theta}\right)^{\frac{1}{\theta}} \cdot L}}$$

$$= \frac{\gamma L}{\left[\delta\left(\frac{L}{K}\right)^{\theta} + (1-\delta)\right]^{\frac{1}{\theta}}}$$

The numerator offers no troubles: but as $\theta \to 0$ the denominator takes intermediate form 1^{α}

$$\text{Let } T = \left[\delta\left(\frac{L}{K}\right)^{\theta} + (1-\delta)\right]^{\frac{1}{\theta}}$$

$$\therefore \qquad \log \quad T = \frac{1}{\theta}\log\left[\delta\left(\frac{L}{K}\right)^{\theta} + (1-\delta)\right]$$

Which is an intermediate from of the class 0/0 as $\theta \to 0$

Applying L'Hopitals Rule:-

$$\lim_{\theta\to 0} \log T = \lim_{\theta\to 0} \frac{\delta e^{\log\frac{L}{K}} . \log\frac{L}{K}}{\left[\delta\left(\frac{L}{K}\right)^{\theta} + (1-\delta)\right]}$$

$$= \delta \log \frac{L}{K}$$

$$\text{Hence} \quad \lim_{\theta\to 0} T = \left(\frac{L}{K}\right)^{\delta}$$

$$\text{Hence equation} \quad \lim_{\theta\to 0} Y = \lim_{\theta\to 0} \gamma\left[\delta K^{-\theta} + (1-\delta)L^{-\theta}\right]^{-\frac{1}{\theta}}$$

may be written as :

$$\lim_{\theta \to 0} Y = \frac{\gamma L}{\left(\frac{L}{K}\right)^{\delta}}$$

$$= \frac{\gamma L}{1} \times \frac{K^{\delta}}{L^{\delta}}$$

$$= \gamma L.L^{-\delta} K^{\delta}$$

$$= \gamma K^{\delta} L^{1-\delta}$$

The R.H.S expression is nothing but Cobb-Douglas production function.

Use of CES :

It was Arrow et al. (1961) who first utilized the CES production function expressed in equation (1) for the estimation of constant returns to scale aggregate production functions using cross-country data. Since then, the ordinary CES production function and its variants have been widely applied in both theoretical and empirical production behavior.

2.3.6 Homothetic Production Function

A homothetic production function is a monotonically increasing transformation of a homogeneous function. Thus any homogenous function is homothetic, but homothetic functions are not necessarily homogenous. The literature does not abound with explicit examples of homothetic functions. There are a good many examples of a simple type, namely transcendental functions. The following interesting form has been suggested by Miss Clemhout (1964):

$$Q = \alpha e^{\gamma t} x^{\lambda}$$

where α, γ, and λ are positive parameters.

For example, a production function U is homothetic if it can be written as a positive monotonic transformation of a homogeneous function.

For example, **U= α–1/ q_1^{α}** is not a homogeneous function. Actually it is homothetic since $\frac{f1}{f2} = \alpha q_2 / q_1$

Homothetic CES production functions:

Any monotonic transformation of the ordinary CES production function like this:

$$Y = A\left[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right]^{-1/\rho}$$

belong to a class of CES production functions called the homothetic class, that is,

Y=F(f), F'>0

where

$$f = A\left[\beta K^{-\rho} + (1-\beta)L^{-\rho}\right]^{-1/\rho}$$

2.4. Isoquants and its Properties

An isoquant can be defined as a curve showing the various combinations of capital and labour required to produce a given quantity of a particular product, in the most efficient way. Isoquans (as opposed to indifference curves) specify cardinal measures of output.

It is the firm's counterpart of the consumer's indifference curve. It is the locus of all combinations of K and L which yield a specified output level. For a given output level, the equation q = f(K, L) becomes q^0 = f(K, L)where q^0 is a parameter.

The locus of all the combinations of K and L which satisfy (1) forms an isoquant. Since the production function is continuous, an infinite number of input combinations lie on each isoquant. All the input combinations which lie on an isoquant will result in the output indicated for that curve.

Within the relevant range of operation an increase of both inputs will result in an increased output. The further an isoquant lies from the origin, the greater the output level which it represents: $Q_3 > Q_2 > Q_1$ (Fig. 2.2)

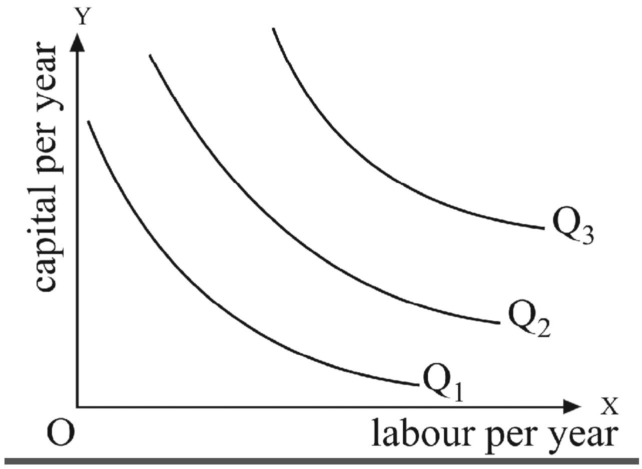


Fig: 2.2 Isoquant

The slope of the tangent to a point on an isoquant is the rate at which K must be substituted for L (or L for K) in order to maintain the corresponding output level. The negative of the slope is defined as the marginal rate of technical substitution (MRTS) :

The MRTS for the firms is analogues to the marginal rate of commodity substituion (MRCS) for the consumer. The MRTS at any point is the same for movements in either direction.

Now, the total differential of the production function is $dq = f_1dL + f_2dK$

Along an isoquant dq = 0, then $0 = f_1dL + f_2dK$

$\therefore$ Therefore the slope of the isoquant = $MRTS_{LK} = -\Delta K/\Delta L = -dK/dL$

Properties of Isoquants

The properties of Isoquant or Iso-product curves are summarized below :

1. Iso-Product Curves Slope Downward from Left to Right :

They slope downward because MTRS of labour for capital diminishes. When we increase labour, we have to decrease capital to produce a given level of output.

The downward sloping iso-product curve can be explained with the help of the following figures :

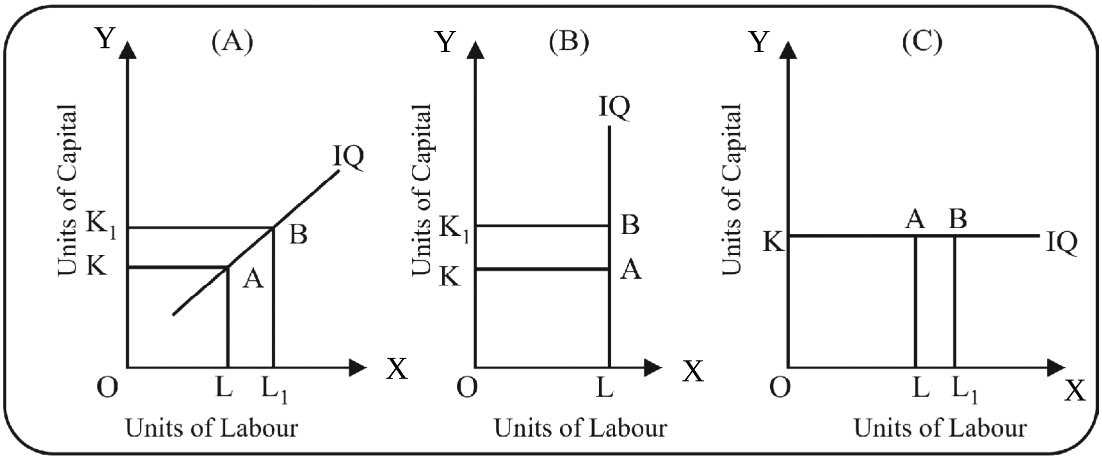


Fig:2.3 Possibilitities of Isoquants

The possibilities of upward sloping, vertical, horizontal curves can be ruled out with the help of the above Figure 2.3 :

(i) The figure (A) shows that the amounts of both the factors of production are increased-labour from L to L_1 and capital from K to K_1. When the amounts of both factors increase, the output must increase. Hence the IQ curve cannot slope upward from left to right.

(ii) The figure (B) shows that the amount of labour is kept constant while the amount of capital is increased. The amount of capital is increased from K to K_1. Then the output must increase. So IQ curve cannot be a vertical straight line.

(iii) The figure (C) shows a horizontal curve. If it is horizontal, the quantity of labour increases, although the quantity of capital remains constant. When the amount of capital is increased, the level of output must increase. Thus, an IQ curve cannot be a horizontal line.

2. Isoquants are Convex to the Origin

Like indifference curves, isoquants are convex to the origin. In order to understand this fact, we have to understand the concept of diminishing marginal rate of technical substitution (MRTS), because convexity of an iso-quant implies that the MRTS diminishes along the isoquant. The marginal rate of technical substition between L and K is defined as the quantity of K which can be given up in exchange for an additional unit of L. It can also be defined as the slope of an isoquant.

It can be expressed as :

$$MRTS_{LK} = -\ \Delta K/\Delta L = -dK/dL$$

Where ΔK is the change in capital and ΔL is the change in labour.

Equation above states that for an increase in the use of labour, fewer units of capital will be used. In other words, a declining MRTS refers to the falling marginal product of labour in relation to capital. To put it differently, as more units of labour are used, and as certain units of capital are given up, the marginal productivity of labour in relation to capital will decline.

This fact can be explained in Fig. 2.4. As we move from point a to b, from b to c and from c to d along an isoquant (IP), the marginal rate of technical substitution (MRTS) of capital for labour diminishes.

Thus it may be observed that due to falling MRTS, the isoquant is always convex to the origin.

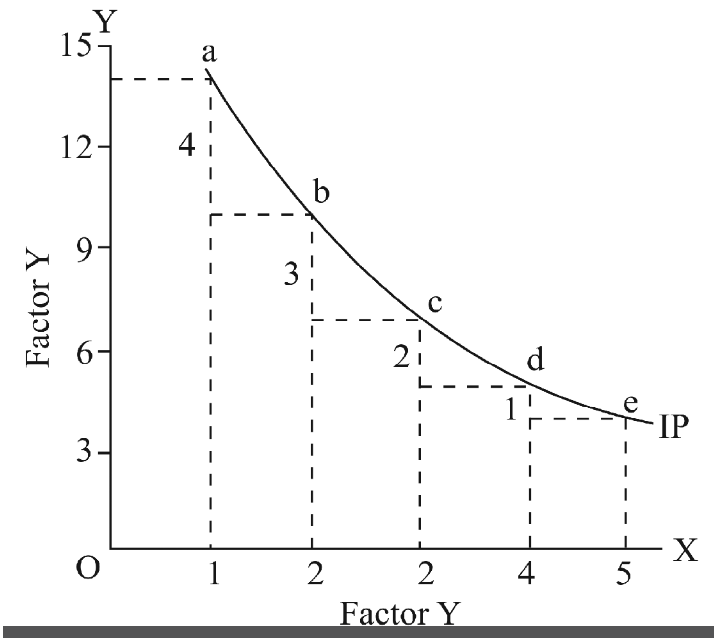


Fig 2.4 : Diminishing MRTS

3. Two Iso-Product Curves Never Cut Each Other :

In Fig. 2.5, two Iso-product curves intersect each other.

Both curves IQ1 and IQ2 represent two levels of output. But they intersect each other at point A. Then combination A = B and combination A = C. Therefore B must be equal to C. This is absurd. B and C lie on two different iso-product curves. Therefore two curves which represent two levels of output cannot intersect each other.

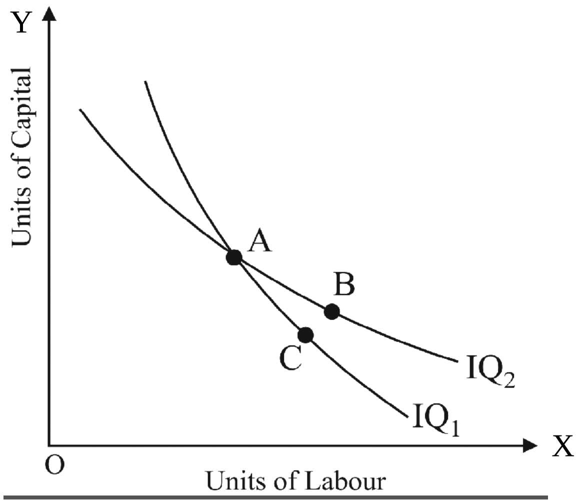


Fig. 2.5 : Two indifference can never intersect

4. Higher Iso-Product Curves Represent Higher Level of Output :

In the Figure. 2.6, units of labour have been taken on OX axis while on OY, units of capital. IQ_1 represents an output level of 100 units whereas IQ_2 represents 200 units of output.

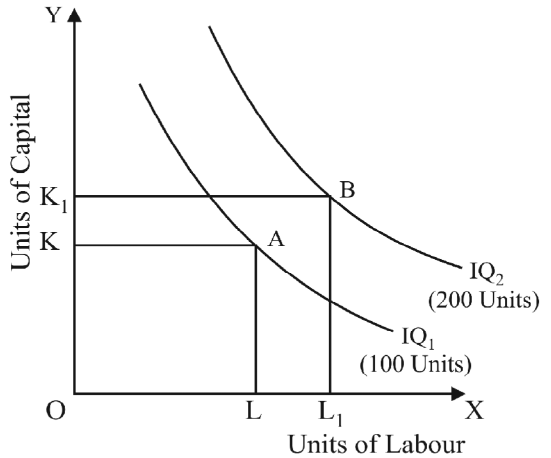


Fig. 2.6 : Higher isoquants

2.4.1 Finding the Optimal Employment of Inputs by Isoquant and Iso-cost lines

We have to know about iso-cost line to find the optimal employment.

An isocost line represents the various combinations of capital and labour firm can buy for a given expenditure.

The present analysis is limited to the case in which the entrepreneur purchases capital K and Labour L in perfectly competitive markets at constant unit prices where the unit price of capital (K) is r_1 and the unit price of labour = r_2. His total cost of production (C) is given by the linear equation

$$C = r_1K + r_2L + b \qquad (1)$$

where r_1 and r_2 are the respective prices of K and L, and b is the cost of any fixed inputs. An isocost line is defined as the locus of input combinations that may be purchased for a specified total cost :

$$C^0 = r_1K + r_2L + b \qquad (2)$$

where C^0 is a parameter.

Solving (2) for K,

The slope of an isocost line equals the negative of the input price ratio .

Condition for Optimization (Graphic Approach)

There will be many combinations of two resources that produce the same level of output. The problem here is to find out that particular combination of inputs, which produces a given quantity of output with minimum cost. Following are the different methods of finding out the least cost combination.

To find out the optimum combination of inputs, through graphic method, both iso-quant and iso-cost line are drawn on the same graph. The slope of the iso-quant indicates the rate of exchangeability (MRTS) between to resources, whereas the slope of iso-cost line represents the inverse price ratio of inputs. The point of tangency between iso-quant and iso-cost indicates the least cost combination (fig. 2.7)

By combining isoquants with isocost it is possible to determine the least-cost process of production. This occurs when isoquant is tangential to the isocost i.e. slope of iso-quant = slope of iso-cost line.

At point e, the slope of the isoquant (IQ_0) is equal to the slope of the isocost line (AB). So, at e we get the least cost combination of inputs. The optimal combination of inputs is given by K* and L*.

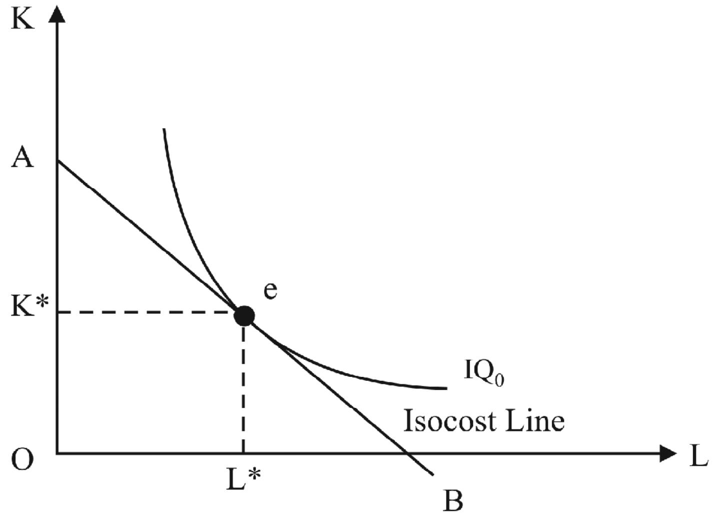


Fig: 2.7 the Least -cost process of production

2.4.2 Returns to Scale

Returns to Scale : Long run laws of production

In the long run, all the inputs are assumed to be variable. Returns to scale means the manner of change in physical output caused by the increase in all the inputs required simultaneously and in the same proportion. Elaborating, suppose one unit of capital and one unit of labour (1K + 1L), produce 100 units of output. Further suppose that both the inputs are doubled, i.e. 2K + 2L (meaning 2 units of capital and 2 units of labour). The point of interest is : will output increase by just 100%; by more than 100%, or by less than 100%. There is no unique answer. All the three states are possible. The three states are respectively called Constant Returns to Scale (CRS), Increasing Returns to Scale (IRS) and Decreasing Returns to Scale (DRS). Let us first illustrate the three states and then explain reasons.

Constant Returns to Scale (CRS): Suppose 1K+1L produce 100 units of output, and 2K+2L produce 200 units of output. It is 100 percent increase in inputs leading to just 100 percent increase in output. This manner of change in output is called CRS. Here average costs stay constant.

Increasing Returns to Scale (IRS): Suppose 1K+1L produce 100 units of output and 2K+2L produce 250 units of output. It is 100 percent increase in inputs in leading to 125 percent increase in output. This manner of change in output is called IRS. This leads to a fall in the average costs and is an example of internal economies of scale.

Decreasing Returns to Scale (DRS): Suppose 1K+1L produce 100 units of output, and 2K+2L produce 180 units of output. It is 100 percent increase in inputs leading to only 80% increase in output. This manner of change in output is called DRS. This leads to an increase in average costs and is an example of internal diseconomies of scale.

Which of the above states actually results depends to a great extent on the type of technology used. There are technologies which result in IRS from the beginning and continue upto a large output level. Similarly, there are technologies leading to CRS almost throughout. There can also be technologies leading to DRS from the very beginning. Besides, it is also possible that a technology is such that it gives IRS in the beginning, followed by CRS and then DRS. For example:

Reasons for IRS:

There are two possible reasons:

More division of labour: Division of labour means subdividing a task into many small sequential operations, with each worker (or a group of workers) assigned each operation. A single worker, instead of doing all the operations, concentrates on only one operation and specializes. This raises efficiency of the worker.

Use of specialized machines: More capital means more capital goods and bigger capital goods. Fully automatic machines can replace the semi-automatic or the hand operated machines. Bigger machines can be used in place of small machines. Bigger capital goods can be used in place of smaller capital goods. It is a common knowledge that a double size capital input may produce more than double the output.

2.5 Cost Function

Before proceeding to our analysis of production cost it is important to be precise about the time period for which the producer's decisions are being analysed. In economics, there are two distinct theoretical time periods: the **short run** and the **long**

run. The short run is defined as a period of time in which the amount of at least one of the factors of production that the producer possesses is fixed. This is most likely to be the case with land, whereas the long run is defined as a time period in which the amounts of all the factors of production that the producer possesses are variable. If the producer wants to increase its floor space then it can do so freely in the long run.

Irrespective of the time period involved, in order to produce anything, the producer has to buy and organize the four factors of production and so necessarily has to pay production cost.

A firm's cost of production include **explicit costs**, (for which it pays out money. Actually it refer to the actual expenditures of the firm to purchase or hire the inputs it needs) and **implicit costs** refer to the value of the inputs owned by the firm and firm used by the firm in its owned production processes, (for which it does not pay money, but for which it nevertheless wants to cover). Explicit costs are wages, intermediate goods (when the firm pays other firms for using many goods and services such as fuel etc.), taxes and interest on loan, while implicit costs include depreciation through wear and tear and obsolescence, etc.

2.5.1 Various Types of cost function

Short run total cost curves:

In the traditional theory of the firm, total cost of the firm in the short run are split into two groups: Total fixed cost (TFC) and total variable cost (TVC).

So, TC = TFC + TVC

Examples of fixed costs: salaries of staff, depreciation of machinery, depreciation of building, land maintence.

Examples of variable costs: raw materials, costs of direct labour, running expenses like fuel cost, repairing cost.

Let us suppose that there are two factors of production: labour (L) and capital (K), where labour is the variable factor and capital is the fixed factor. Let w is the given wage rate and r is the given price of capital services. Then we can write

TFC = r.K

TVC = w.L

TC = wL +rK

Total Fixed Cost Curve: The total fixed cost (TFC) curve is a horizontal line. Total fixed cost does not change with the quantity of output produced, thus the TFC curve is a flat, horizontal line.

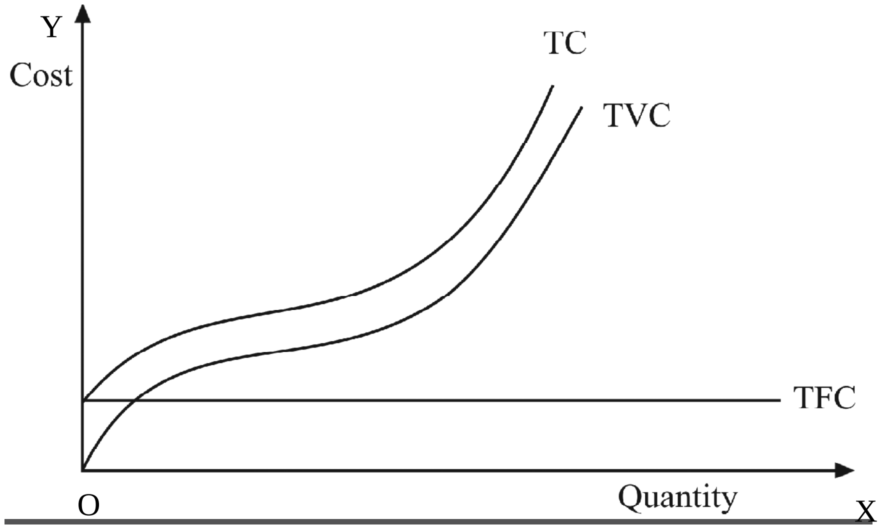


Figure 2.8 Total Cost curves

Total Variable Cost Curve: The total variable cost (TVC) curve is a positively-sloped line that reflects increasing then decreasing marginal returns. The TVC curve emerges from the origin with a relatively steep slope, flattens, then becomes increasingly steeper.

Total Cost Curve: The total cost (TC) curve can be derived as the vertical summation of the TVC and TFC curves. In other words, the TC curve can be found by shifting the TVC vertically by the amount of TFC. This means that the shape of the TC curve is identical to that of the TVC. The two curves have identical slopes for each quantity of output.

The three curves reflecting total fixed cost curve, the total variable cost curve, and the total cost curve are given in figure 2.8 .An important conclusion from this derivation of the TC curve is that the vertical distance between TC and TVC curves is the same at all output quantities. The reason, of course, is that this vertical distance is total fixed cost. Because total fixed cost is constant, the vertical distance is constant.

Short run Average and Marginal Cost Curves:

Average fixed cost (AFC): Average fixed cost is defined as the total fixed cost per unit of output i.e. total fixed cost divided by output.

$$AFC = \frac{TFC}{Q}$$

Average Variable cost (AVC): Average variable cost is defined as the total fixed cost per unit of output i.e. total variable cost divided by output

$$AVC = \frac{TVC}{Q}$$

Average total cost (ATC): Average total cost is defined as the total cost per unit of output ie. Total cost divided by output.

$$ATC = \frac{TC}{Q}$$

Marginal cost: Marginal cost is the addition to total cost attributable to the addition of one unit of output. It is change in total cost divided by change in total output.

$$MC = \frac{change\ in\ TC}{change\ in\ Q} = \frac{change\ in\ TVC}{change\ in\ Q}$$

Or, $$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$$

The Table 5.1 gives an example to work out different types of costs for different levels of output. The table presents a series of output (Q) and corresponding TFC,TVC, TC, AFC, AVC, ATC and MC for output from 0 to 10 units.

AFC Curve: The AFC initially falls strongly but its reduction gets a smaller as output continues to expand. As is seen from the table, as the output increases from 0 to 10, AFC declines throughout. So, the curve will be negatively sloped. Again, AFC× Q =

Table 2.1: Calculation of TC, AFC, AVC, ATC and MC

Output (Q)	Total Fixed Cost (TFC)	Total Variable Cost (TVC)	Total cost (TC)	Average Fixed Cost (AFC)	Average Variable Cost (AVC)	Average Total Cost (ATC)	Marginal Cost (MC)
0	**12**	**0**	**12**	**-**	**0**	**-**	**-**
1	12	28	40	12	28	40	28
2	12	44	56	6	22	28	16
3	12	54	66	4	18	22	10
4	12	62	74	3	15.5	18.5	8
5	12	68	80	2.4	13.6	16	6
6	12	75	87	2	12.5	14.5	7
7	12	84	96	1.7	12	13.7	9
8	12	100	112	1.5	12.5	14	16
9	12	132	144	1.3	14.7	16	32
10	12	178	190	1.2	17.8	19	46

TFC = constant. So, mathematically, the AFC curve is a rectangular hyperbola (Figure 2.9).

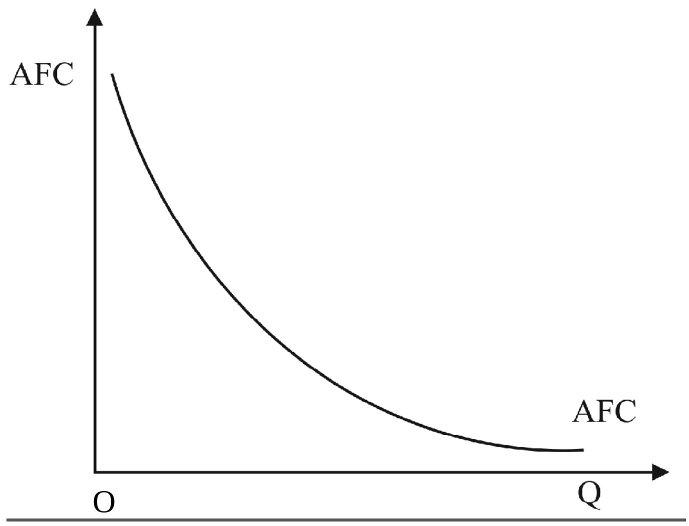


Fig. 2.9 Average Fixed Cost (AFC) Curve

AVC Curve: AVC is U-shaped (Fig 2.9). It first declines, reaches a minimum and then rises thereafter. The reason for U-shaped nature of AVC lies in the theory of production.

$$AVC = \frac{TVC}{Q} = \frac{W \times L}{Q} = \frac{W}{Q/L} = \frac{W}{APL}$$

Thus AVC is the price per unit of input multiplied by the reciprocal of the average product. Since, the average product normally rises, reaches a maximum and then declines, the AVC normally falls, reaches a minimum and then rises thereafter.

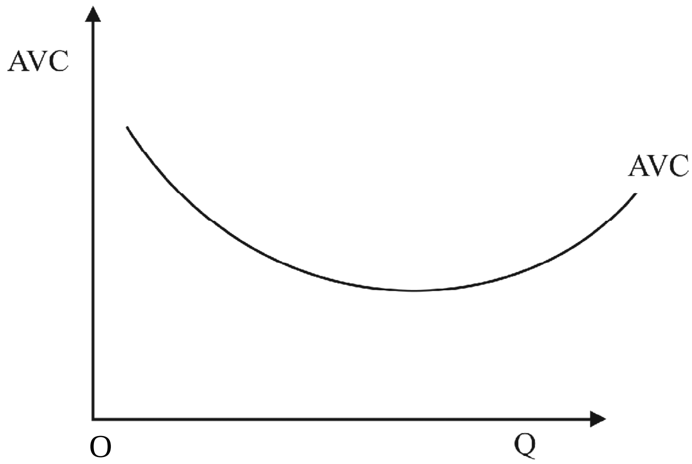


Fig. 2.10 Average Variable Cost (AVC) Curve

ATC Curve:

In the short run we have already seen the following relation:

$$TC = TFC + TVC$$

Or, $$\frac{TC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q}$$

Or, $$ATC = AFC + AVC$$

AVC curve is also U-shaped (Fig 2.10). These two effects logically explain the shape. Short run average total cost initially falls as output expands because at first the average fixed cost and the average variable cost both declines; but, as output expands further, at some point the average variable cost effect begins to outweigh the average fixed effect, causing short run average cost to rise again.

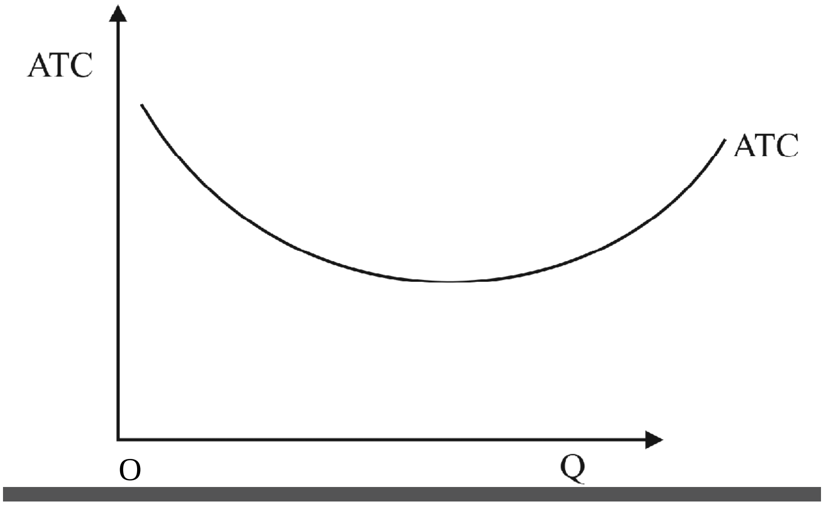


Fig. 2.11 Average Total Cost (ATC) Curve

Marginal Cost: MC curve is also U-shaped (Fig 2.12). It first declines, reaches a minimum and rises thereafter. The explanation for this curvature lies in the theory of production.

$$MC = \frac{\Delta TVC}{\Delta Q} = \frac{\Delta(W \times L)}{\Delta Q} = \frac{\Delta W}{\Delta Q / \Delta L} = \frac{W}{MPL}$$

Here w is fixed as the entrepreneur is perfect competitor in the input market. Thus MC is the price per unit of input multiplied by the reciprocal of the marginal product.

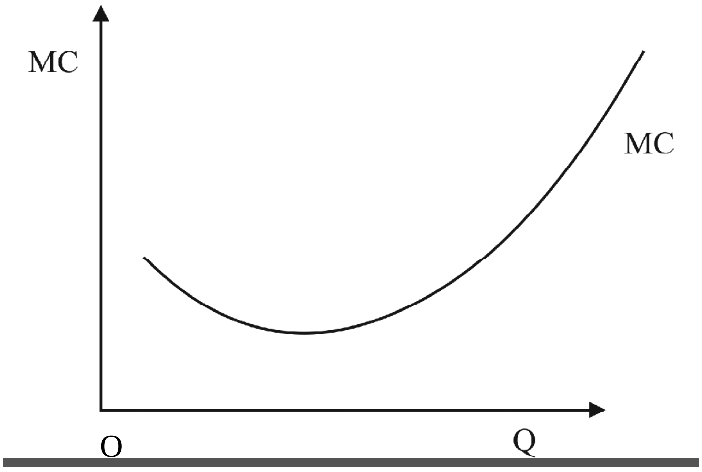


Fig 2.12 Marginal Cost (MC) Curve

Since, the margianal product normally rises, reaches a maximum and then declines, the MC normally falls, reaches a minimum and rises thereafter.

Table 2.2 summarises many types of costs.

Table 2.2: Various Types of Costs: A Summary

Term	**Definition**	**Mathematical Description**
Explicit costs	Costs that require an outlay of money by the firm	
Implicit costs	Costs that do not require an outlay of money by the firm	
Fixed costs	Costs that do not vary with the quantity of output produced	FC
Variable costs	Costs that vary with the quantity of output produced	VC
Total cost	The market value of all the inputs that a firm uses in production	TC = FC + VC
Average fixed cost	Fixed cost divided by the quantity of output	AFC = FC/Q
Average variable cost	Variable cost divided by the quantity of output	AVC = VC/Q
Average total cost	Total cost divided by the quantity of output	ATC = TC/Q
Marginal cost	The increase in total cost that arises from an extra unit of production	MC = DTC/DQ

Relationships among AC, AVC, AFC and MC

The relation among AFC, AVC, ATC and MC is shown in the figure 2.13.

(i) AFC declines continuously, approaching both axes asymptotically as shown by points 1 and 2 in the figure. AFC is a rectangular hyperbola.

(ii) AVC first declines, reaches a minimum at point 4, and rises thereafter. When AVC attains its minimum at point 4, MC=AVC. As AFC approaches asymptotically close to the horizontal axis, AVC approaches ATC asymptotically as shown by point 5.

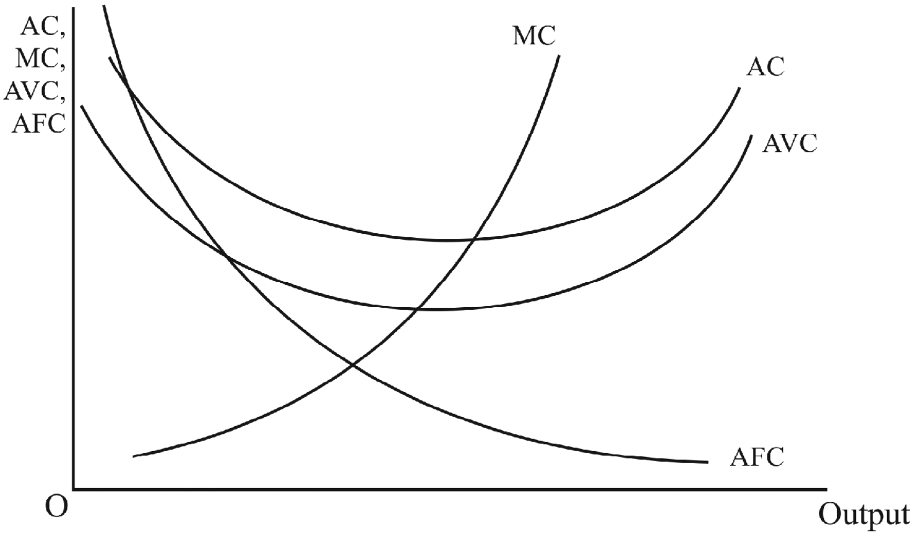


Fig 2.13 : Relationships among AC, AVC, AFC and MC

(iii) ATC first declines, reaches a minimum at point 3, and rises thereafter. At the minimum of ATC (at point 3), MC=ATC

(iv) MC first declines, reaches a minimum at point 6 and rises thereafter. At the minimum points of both AVC and ATC, MC equals both i.e., MC= AVC at minimum point of AVC and MC = ATC at minimum point of ATC.

(v) When AVC and ATC fall, MC lies below both of them. When AVC and ATC rise, MC lies above both of them.

2.5.2 Relationship between Short- Run Cost and Long-Run Cost Curves

In the long run all factors are assumed to become variable. The long run cost curve is a planning curve in the sense that it guides the entrepreneurs in their decisions to plan the future expansion of their output.

The long run average cost (LAC) curve is derived from short run cost curves. Each point on the LAC corresponds to a point on a short run average cost (SAC) curve, which is tangent to the LAC at the point. The long-run average cost curve is the envelope of an infinite number of short-run average total cost curves, with each short-run average total cost curve tangent to, or just touching, the long-run average cost curve at a single point corresponding to a single output quantity. It can be said that an economic agent operates in the short run and plans in the long run.

Let us examine how the LAC is derived from the SAC curves.

2.5.3 Derivation of LRAC curve from SRAC curve

Assume that the available technology to the firm at a particular point in time includes three methods of production, each corresponding to different plant size – small plant, medium plant and large plant. The average cost curve for the small plant is denoted by SAC1, for second plant by SAC2 and the third plant by SAC3 (Fig 2.14). If the firm plans to produce X1, the firm will choose SAC1. If the firm plans to produce X2, the firm will choose SAC2. If the firm plans to produce X3, the firm will choose SAC3. Upto the output level X1//, as the demand rises firm will continue to produce to produce with small plant. If the firm expects that, future demand will rise beyond X1//, it will install medium plant (denoted by SAC2) because the output can be produced at a lower cost. Similarly the firm will produce with large plant beyond X2//. In the short run, the firm must operate with SAC1, SAC2 or SAC3. But in the long run it is possible to build whose size leads to the least average cost for any output. Thus as a planning device, the heavily shaded curve is the long run average cost curve because this curve shows the long run average cost of producing each possible output. This curve is frequently called the envelope curve.

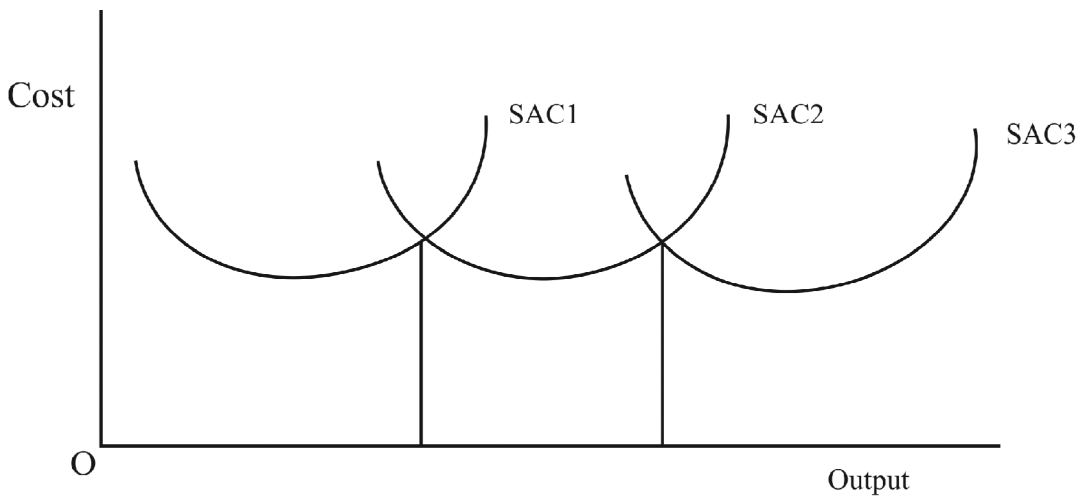


Fig 2.14: Derivation of LAC Curve from SAC Curves

Now, if we assume that there is a large number (infinite number) of plants, we obtain a continuous curve which is the planning LAC curve of the firm. Each point of this curve shows the minimum (optimal) cost of producing the corresponding level of output. Thus, the LAC curve is the locus of points denoting the least cost of producing the corresponding output. It is called planning curve on the basis of this curve, the firm decides what plant is to set up in order to produce optimally (at minimum cost) the expected level of output. It is also called envelope curve because it envelops the short run average cost curves.

2.5.4 Economies of Scale & Diseconomies of Scale

Economies of Scale:

Economies of scale exist when the expansion of all inputs, especially labo rand capital, result in a decrease in long-run average cost (Fig 2.15). Economies of scale result from increasing returns to scale and are graphically illustrated by a negatively-sloped long-run average cost curve. Economies of scale usually occur for relatively small levels of production and are then overwhelmed by diseconomies of scale for relatively large production levels. Together, economies of scale and diseconomies of scale create a U-shaped long-run average cost curve.

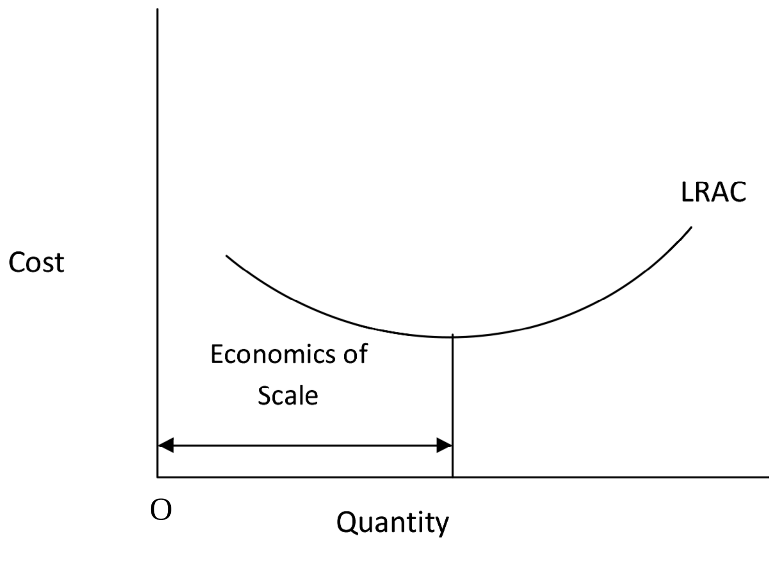


Fig 2.15 Economies of Scale

Reasons for Economies of Scale:

Economies of scale are the result of: (1) increased resource specialization, (2) decreased resource prices, (3) increased bi-product use, (4) increased auxiliary activities, and (5) the geometric relation between volume and area.

i) **Increased Resource Specialization:** As an activity expands, production activities are often divided into distinctive specialized tasks that can be performed by specialized resources. With a small scale of production, one worker might use a simple sewing machine. However, larger scale production might involve several different types of sewing machines Workers are also likely to be specialized in the operation of each machine.

ii) **Decreased Resource Prices:** Another advantage of expanding the scale of production is lower resource prices. A bigger activity might, for example, receive volume discounts from suppliers. These suppliers might be able to provide discounts because they are more efficiently using their fixed inputs in the short run.

iii) **Increased Bi-product Use:** Most production activities generate bi-products. In many cases these are waste residuals tossed out with the trash. Some of these bi-products, however, can be valuable if the quantity is large enough. Larger scale production often generates sufficient quantities of bi-products to make them worth marketing and selling.

Diseconomies of Scale:

Diseconomies of scale exist when the expansion of all inputs, especially labor and capital, result in an increase in long-run average cost (Fig 2.16). Diseconomies of scale result from decreasing returns to scale and are graphically illustrated by a positively-sloped long-run average cost curve. Diseconomies of scale usually occur for relatively large levels of production and overwhelm economies of scale that occurs at relatively small production levels. Together, economies of scale and diseconomies of scale create a U-shaped long-run average cost curve.

Diseconomies of scale are the result of: (1) decreased management control and (2) increased resource prices.

i) **Decreased Management Control:** As the scale of operation expands, control over production tends to decline. This is often seen by an increase in the number of layers of managers and an increased separation between owners and workers.

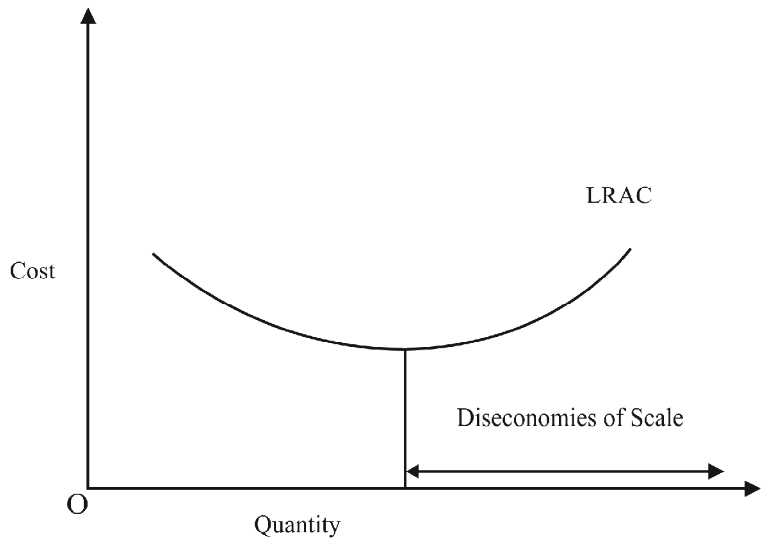


Fig 2.16 Diseconomies of Scale

ii) Increased Resource Prices: As the scale of production increases from small to large, some resource prices are likely to decline as suppliers provide volume discounts and take advantage of their own decreasing average cost. However, with additional expansion of the scale, resource prices might very well increase. Higher resource prices raise average cost.

Limit of economies of scale: When economies of scale reach their limit and diseconomies of scale are yet to begin, the returns to scale become constant. It will happen when factors of production are perfectly divisible and where technology is such that capital- labour ratio is constant.

Why are the Short Run average cost curve and Long Run Average Cost curves U- shaped?

The short run and long run average cost curves are both U-shaped. However the reasons for this shape are quite different.

Short run average cost (SAC) curve is U-shaped because the decline in average fixed cost (AFC) is ultimately more than offset by the rise in average variable cost (AVC). The shape of AVC is U- shaped because the AP reaches a maximum and declines because of the operation of law of diminishing returns to the variable factor.

Economies of Scale: In the long run, there are no fixed inputs. As such, the law of diminishing marginal returns does not operate and marginal returns do not guide production in the long run. The LAC is U-shaped because of the following reasons. The decreasing portion of LAC is caused by increasing returns to scale (including financial economies) and the increasing portion of the LAC is caused by decreasing returns to scale (including financial diseconomies).

2.5.5 The Learning Curve

A learning curve is a concept that graphically depicts the relationship between the cost and output over a defined period of time, normally to represent the repetitive task of an employee or worker. The learning curve was first described by psychologist Hermann Ebbinghaus in 1885 and is used as a way to measure production efficiency and to forcast cost.

In the visual representation of a learning curve, a steeper slope indicates initial learning translates into higher cost savings, and subsequent learnings result in increasingly slower, more difficult cost savings.

The learning curve also is referred to as the experience curve, the cost curve, the efficiency curve, or the productivity curve. This is because the learning curve provides measurement and insight into all the above aspects of a company. The idea behind this is that any employee, regardless of position, takes time to learn how to carry out a specific task or duty. The amount of time needed to produce the associated output is high. Then, as the task is repeated, the employee learns how to complete it quickly, and that reduces the amount of time needed for a unit of output.

That is why the learning curve is downward sloping in the beginning with a flat slope toward the end, with the cost per unit depicted on the Y-axis and total output on the X-axis. As learning increases, it decreases the cost per unit of output initially before flattening out, as it becomes harder to increase the efficiencies gained through learning.

2.5.6 Application of Duality Approach

Duality approach can be succinctly applied to consumer theory which is written as follows:

The Application of Cost Functions to Consumer Theory

The cost function and production function framework described in the previous sections can be readily adapted to the consumer context from the duality approach: simply replace output y by utility u, reinterpret the production function F as a utility function, reinterpret the input vector x as a vector of commodity demands and reinterpret the vector of input prices p as a vector of commodity prices. With these changes, the producer's cost minimization problem becomes the following problem of minimizing the cost or expenditure of attaining a given level of utility u: $C(u, p) \equiv$ minx $\{pT\ x : F(x) \geq u\}$. If the cost function is differentiable with respect to the components of the commodity price vector p, then Shephard's (1953; 11) Lemma applies and the consumer's system of commodity demand functions as functions of the chosen utility level u and the commodity price vector p, x(u,p), is equal to the vector of first order partial derivatives of the cost or expenditure function C(u,p) with respect to the components of p:

$$x(u, p) = \nabla pC(u, p)$$

The demand functions xn(u,p) defined in the above equation are known as Hicksian demand functions. Thus it seems that we can adapt the theory of cost and production functions used in section 2 above in a very straightforward way, replacing output y by utility u and reinterpreting. ——

2.6 Market Structure

Market is a place or institution in which buyers and sellers of a good or asset exchange. Markets facilitate trade in goods, as in commodity markets; in securities, for example, the bond market, the capital market, or the stock exchange, in labour services, as in the labour market; or in foreign exchange, in the foreign exchange market. Market is an exchange mechanism that brings together sellers and buyers of a product.

The theory of markets distinguishes between markets according to their structural characteristics, in particular the number of sellers and buyers involved. A number of 'market' situations can be identified like perfect competition, monopoly etc. in Table 2.3.

Table 2.3: Market Morphology

Perfect Competition or Competitive Market	=	Many sellers, many buyers
Monopoly	=	Mono(One), Poly (seller)
Bilateral Monopoly	=	One seller (like monopoly) and one buyer (like Monopsony)
Duopoly	=	Two sellers, many buyers
Monopsony	=	Many sellers, one buyer
Duopsony	=	Many sellers, two buyers
Oligopoly	=	Few sellers, many buyers
Bilateral Oligopoly	=	Few sellers, few buyers
Oligopsony	=	Many sellers, few buyers

2.6.1 Perfect Competition

Perfect competition or completive market is a market structure characterized by large numbers of buyers and sellers for a product. For example, in some labour markets, the labour is hired by many firms and is supplied by many households: a labour market like this is called competitive labour market.

A. Assumptions (of Perfect Competition)

Perfect competition is imbibed with the following assumptions:.

(i) **A large number of sellers and buyers.** Under perfect competition, the number of sellers and buyers is very large. The number of seller is so large that the share of each seller in total supply of a product is too small for a single seller to affect the market price by changing his supply. Likewise, the number of buyers is so large that the share of each buyer in total demand is too small for a single buyer to influence the market price by changing his/her demand.

(ii) **Homogeneous products.** Products supplied by all firms are almost homogeneous. Homogeneity of products means that products supplied by various firms are so identical in appearance and use that buyers do not distinguish between them nor do they prefer the product of one firm to that of another. For example, wheat the vegetables produced by all the farmers, other things given, are treated as homogeneous.

(iii) **Free entry and free exit of firms.** There is no barrier, legal or market-related, on the entry of new firms into or exit of existing one from the industry. Firms are free to enter the industry and quit it at their free will.

(iv) **Absence of collusion or artificial restraint.** There is no sellers' union or other kinds of collusions between the sellers such as cartels or guilds, nor is there any kind of collusion between the buyers, *e.g.,* consumers' associations or consumer forum. Each seller and buyer acts independently.

(v) **No government intervention.** In a perfectly competitive market, there is no government intervention with the working of the market system. There is no licencing system regulating the entry of firms to the industry, no regulation of market prices. i.e., fixation of lower or upper limits of prices, no control over the supply of inputs, no fixation of quota no production, and no rationing of consumer demand, no subsidy to producers or to consumers, etc.

(vi) **Perfect mobility of factors of production.** For a market to be perfectly competitive there should be perfect mobility of resources. This means that the factors of production must be in a position to move freely into or out of an industry and from one firm to another.

(vii) **Perfect knowledge.** There is perfect dissemination of the information about the market conditions. Both buyers and sellers are fully aware of the nature of the product, is availability or saleability and of the price prevailing in the market.

Perfect competition**,** as characterized above, is an uncommon phenomenon in the real business world. However, the actual markets that approximate to the conditions of perfectly competitive model include the share markets, securities and bond markets and agricultural product markets, e.g., local vegetal markets. Sometimes a distinction is made between perfect competition and pure competition. The difference between the two is only a matter of degree. **Perfect competition** less perfect mobility of factors of production and perfect knowledge is regarded as **pure competition.**

B. The Perfectly Competitive Firm as a Price Taker

In a perfectly competitive market there are many firms. One firm's output decisions cannot influence the overall market supply to any noticeable extent. If one firm changes its output level then this has such a small effect on the industry supply that the market price does not alter.

Each firm is therefore a 'price taker'; it is so small that its actions cannot influence the market price. The firm can sell as much as it wants without bringing down the market price. This means that every unit can be sold at the market price. For example, every unit can be sold at Rs.10, so the extra revenue generated from a sale is the same as its price. This means that the marginal revenue is the price (P=MR), as shown in Figure 2.17.

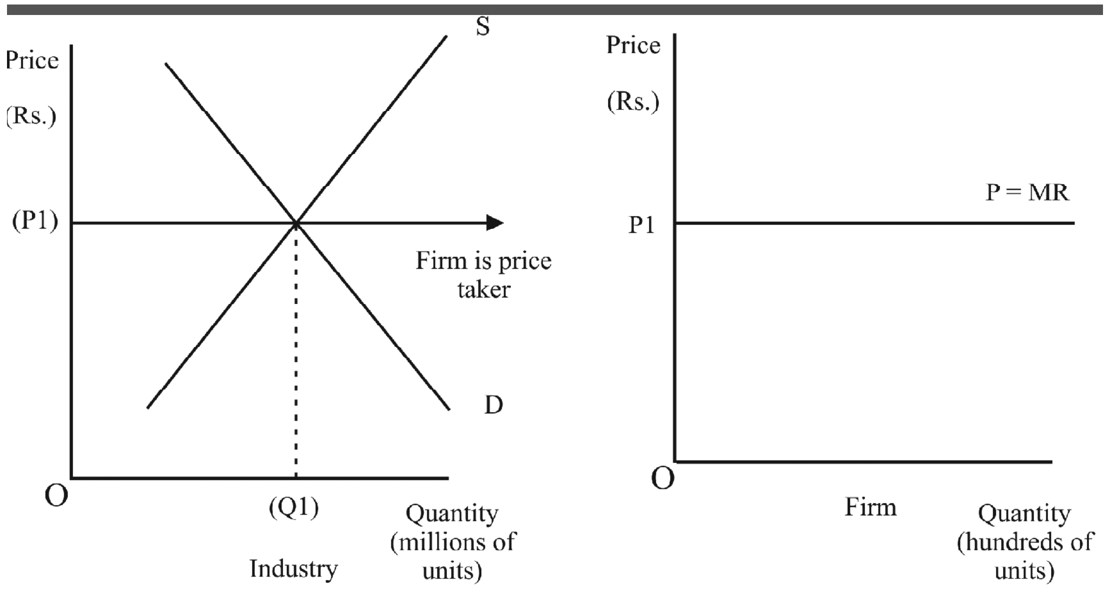


Fig 2.17: Perfectly Competitive Firm As a Price Taker

C. Short-run Equilibrium (of a firm) under perfect competition

In the short run firms in perfect competition are able to make abnormal profits (when t he price is greater than the average cost) or losses (when the price is less than the average cost). However, this situation will not continue in the long run.

If firms are making abnormal profits (Fig 2.18) then this acts as a signal for other firms to enter the market to benefit from this. The entry of more firms will lead to more being supplied and will shift the industry supply curve to the right; this will reduce the market price. (Although one firm cannot shift the industry supply on its own, the entry of many firms will shift the curve to the right.) This process will continue until only normal profits are being made (the price equals the average cost). When normal profits are being made there is no incentive for more firms to enter or leave the industry.

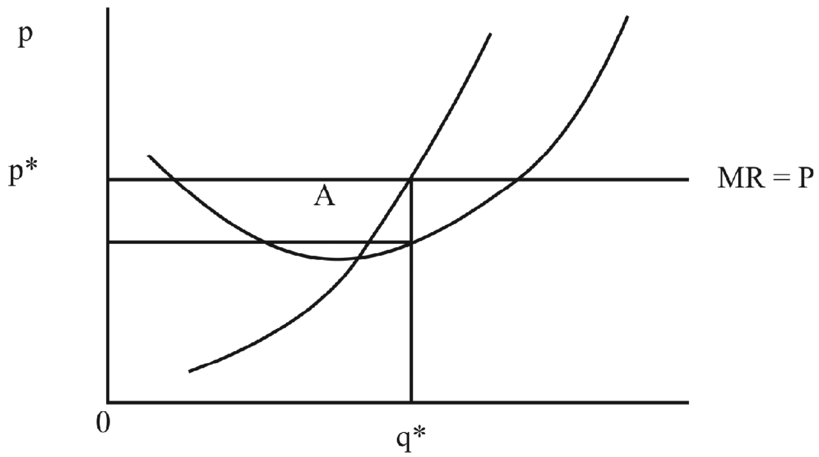


Fig 2.18. Equilibrium of a firm under perfect competition (Supernormal Profit)

If firms are making losses (Fig.2.19) then this means that businesses will leave the industry. This shifts the industry supply curve to the left and increases the market price. This will continue until only normal profits are being made.

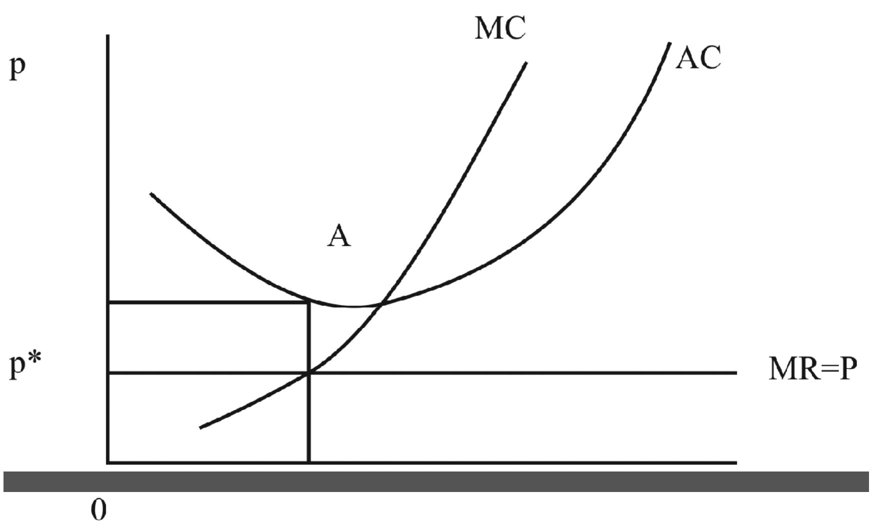


Fig 2.19. Equilibrium of a firm under perfect competition (Loss)

Profit Maximization and Short- Equilibrium under Perfect Competition using Total Revenue, Total Cost and Profit curve

Total profit is an important concept because it is what motives the firms that supply goods and services. The assumed goal of the firm is to maximize profit. The profit of a firm measures the difference between the value of what has been sold and the cost of the production to produce these goods.

Total Profit (ð) = Total Revenue (TR) – Total Cost (TC)

or, $\pi = TR - TC$

Profit Maximization:

The process of obtaining the highest possible level of profit through the production and sale of goods and services is called profit maximisation. The profit-maximization assumption is the guiding principle underlying production by a firm. Profit is the difference between the total revenue received from selling output and the total cost of producing that output. The profit-maximization assumption means that firms seek a production level that generates the greatest difference between total revenue and total cost. If a firm maximizes profit, then it is generating the highest possible reward.

Profit Maximization Conditions

Profit-maximizing output can be identified in one of three ways—directly with economic profit, with a comparison of total revenue and total cost, and with a comparison of marginal revenue and marginal cost.

The top panel presents the profit curve. The middle panel presents total revenue and total cost curves. The bottom panel presents marginal revenue and marginal cost curves

Profit: First, profit maximization can be illustrated with a direct evaluation of profit. If the profit curve is at its peak, then profit is maximized.

1. Total Revenue and Total Cost: Second, profit maximization can be identified by a comparison of total revenue and total cost. The quantity of output that achieves the greatest difference of total revenue over total cost is profit maximization. In the middle panel, the vertical gap between the total revenue and total cost curves is the greatest. For smaller or larger output levels, the gap is either less or the total cost curve lies above the total revenue curve.

2. Marginal Revenue and Marginal Cost: Third, profit maximization can be identified by a comparison of marginal revenue and marginal cost. If marginal revenue is equal to marginal cost, then profit cannot be increased by changing the level of production. Increasing production adds more to cost than revenue, meaning profit declines. Decreasing production subtracts more from revenue than from cost, meaning profit also declines. In the bottom panel, the marginal revenue and marginal cost curves intersect. At larger or smaller output levels, marginal cost exceeds marginal revenue or marginal revenue exceeds marginal cost.

Before dealing with the concept of profit maximization it is important to note a number of points from Fig 2.20.

The Marginal Rule for Profit Maximisation

The maginal rule for profit maxmisation states that profit maximizing output occurs when when marginal revenue equals marginal costs, that is MR =MC

If, MR>MC, then extra unit will make a profit. This means that by selling extra unit, the total profit will go up.

If, MR<MC, then extra unit will make a loss. This means that by selling extra unit, the total profit will go down. So, these units should not be produced.

Producers equilibrium

A perfectly competitive firm is presumed to produce the quantity of output that maximizes economic profit—the difference between total revenue and total cost. This production decision can be analyzed directly with economic profit, by identifying the greatest difference between total revenue and total cost, or by the equality between marginal revenue and marginal cost.

D. Shut down point and Break even point

Shut down Point

So far, we have been analyzing the question of how such a competitive firm will produce. In certain circumstances, however, the firm will decide to shut down and not product anything at all.

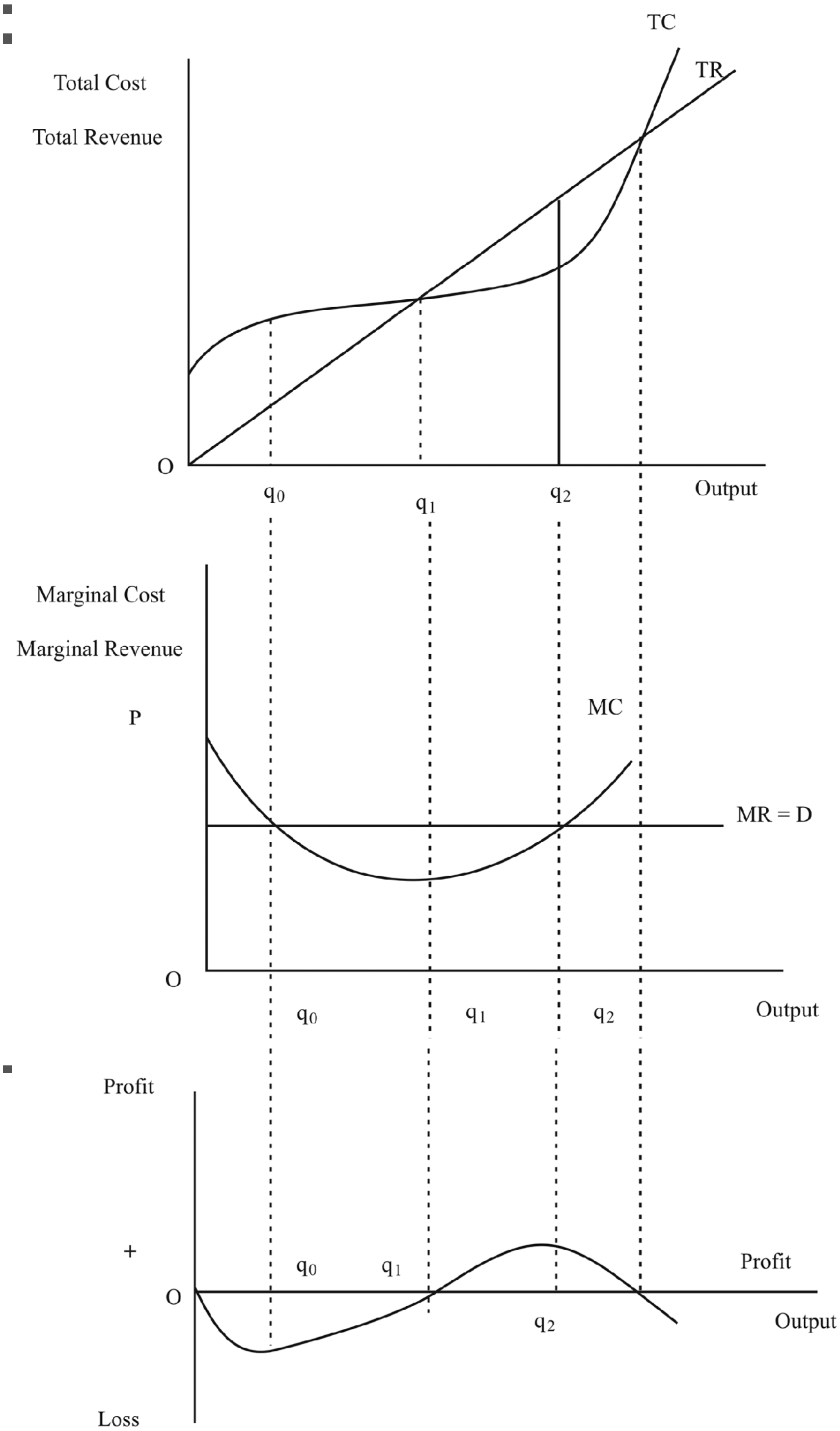


Fig 2.20: Derivation of profit Maximizing Firm

If the firm stops producing, then its revenue will be 0 and its short run variable cost will be 0. But its fixed cost will remain. The firm will continue to operate in the short run if total revenue (TR) exceeds total short run variable cost (TVC).

Loss if the firm stays in business = TC-TR

Loss if the firm shuts down = TC – TVC

The firm will shut down if the firm's loss if it stays in business is greater than its loss if the firm shuts down (Fig 2.20):

TC-TR > TC – TVC

Or, -TR > – TVC

Or, TR< TVC

Or, P.Q < AVC.Q

Or, P<AVC

So, the firm will shut down in the short run, if the price (determined by the interaction of market demand and market supply) is less than average variable cost.

If, AVC<P<AC, then firm will operate even if the firm loses.

Break Even Point

The output level at which the firm's TR equals its TC and the firm's total profits are zero. That is at A, we have Total cost equal to total revenue so profit is basically zero.

The shut down and break even points are shown in Figure 2.21)

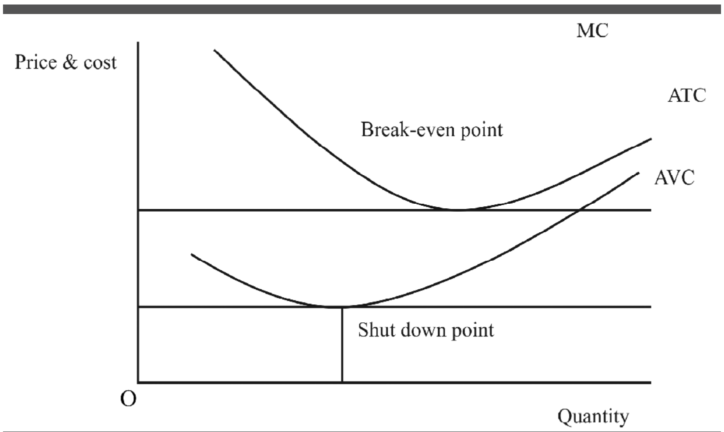


Fig. 2.21. Shut Down and Break even Point

E. Supply curve of the firm under perfect competition

Supply Curve of the firm:

The short run supply curve of a firm in perfect competition is precisely its marginal cost curve for all output levels with P>AVC. For market prices lower than minimum average variable cost (AVC), equilibrium quantity supplied is zero.

The Supply Curve of a firm:

The supply functions of individual firms can be defined for (1) a very short period during which output level cannot vary, (2) a short run during which output level can be varied but plant size cannot, and (3) a long run in which all inputs are variable.

The Very Short Period

Assume that the entrepreneur decides every morning how much to produce that day. His output decision is instantly implemented, and he spends the rest of the day trying to sell his output at the highest possible price. He cannot increase his output during the day and sells a given stock of the commodity. Since an output q^0 has already been produced, the marginal cost of any output less than q^0 is zero. Output cannot be increased beyond this point in the very short period, and the marginal cost of higher outputs may be considered infinite. The marginal cost curve is represented by a vertical line at this point.

The firm maximizes profit by selling a quantity for which MC = p. Since the MC of any output less than q^0 is zero and the MC of any output greater than q^0 is infinite, the equality MC = p cannot be satisfied, and the firm will expand sales to the point at which price ceases to exceed MC. Therefore, it will sell its entire output (i.e., its entire stock of the commodity) at the prevailing price. This maximizes profit, because the prevailing price is the highest price at which the output can be sold. Quantity sold does not respond to price changes. In general, the aggregate supply function states the quantity that will be supplied by all producers as a function of the price. Since the output of each firm is fixed, the aggregate supply of the commodity is also given and does not depend upon the price. The supply curve is a vertical line.

The Short Run

The firm's short-run supply curve is identical with that portion of its short-run MC curve which lies above its AVC curve. The firm's supply curve consists of the segments OA and BC in Fig 2.22.

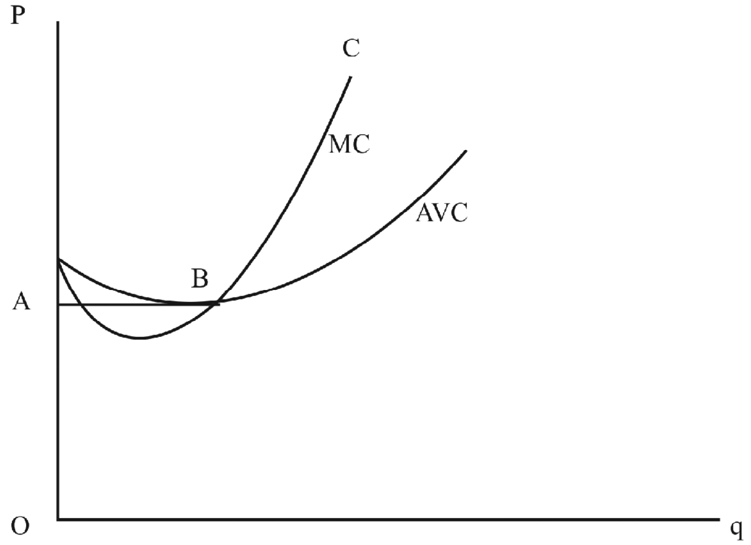


Fig 2. 22. : Supply Curve of a firm under perfect competition in the short-run

The ith firm's short-run MC is a function of its output:

$$MC_i = \Phi'_i(q_i) \qquad (1)$$

The supply function of the ith firm is obtained from its first-order condition for profit maximization by letting p = MC and solving (1) for qi = Si:

$S_i = S_i(p)$ for p>= min AVC

Si = 0 for p < min AVC

The aggregate supply function for Q is obtained by summing the n individual supply functions. The aggregate supply is:

$$S = \sum_{i=1}^{n} S_i(p) = S(p)$$

The aggregate supply curve is the horizontal sum of the individual supply curves.

The Long Run

The firm's long-run optimal output is determined by the equality of price and long-run MC. Zero output is produced at prices less than AC, and the firm's long-run supply function consists of that portion of its long-run MC function for which MC exceeds AC. The mathematical derivation of the long-run aggregate supply function is similar to the derivation of the short-run supply function. The MC function of the ith firm is

$$MC_i = \Phi'_i(q_i) \qquad i = 1, \ldots, n$$

Setting $p = MC_i$ and solving for $q_i = S_i$

$$S_i = S_i(p) \qquad i = 1, \ldots, n \qquad (2)$$

The aggregate supply function is then obtained by adding the n individual supply function in (2). In the absence of external effects the long-run supply function is positively sloped for the same reason as the short-run supply function.

The supply curve of the industry: The shape of the industry's supply curve depends on the length of time available for quantity to respond to price. We will consider first the case in which the period is short enough so that the number of firms can be taken as fixed and then the case in which the period is long enough to permit free entry and exit.

No entry or Exit:

With the number of firms in the industry fixed, the industry's supply curve can be derived by simply summing the quantities supplied by individual firms. If there are n firms whose individual supplies are $q^1(p)$, ….. $q^n(p)$, the industry supply is

$$Q(p) = \sum_{i=1}^{n} q^i(p)$$

On particularly simple special case is that in which all n firms are identical. In this instance, the industry supply is just a multiple of a firm's supply: $Q(p) = nq(p)$. In this case it is clear that the shape of the industry supply curve is qualitatively similar to that of a firm's supply curve: vertical for prices below minimum average cost and positively sloped for higher prices.

The supply curve for the industry is the sum (or aggregate) of all the supply curves of the individuals firms. At each and every price the amount that each firm is willing and able to supply can be added together to give the quantity supplied by the industry.

F. Long run Equilibrium under industry in perfect competition

The long-run equilibrium in perfect competition is shown in Fig.2.23. In the long run in perfect competition firms can only make normal profits. Assuming they are profit maximisers, they will produce when the marginal revenue equals the marginal costs (MR = MC).

Given that the firm is a price taker, the marginal revenue will equal the price (P = MR). This means that firms will produce when the price, the marginal revenue and the marginal cost are all equal. As a result, firms will be allocatively efficient. Allocative efficiency occurs when the extra benefit to society (as shown by the price that consumers are willing to pay) equals the extra costs, that is, the price equals the marginal cost. In the long-run equilibrium of perfect competition, firms are producing all of the units for which the price (which represents the extra benefit or utility to the consumer) is greater than the extra cost of producing it, up to the point where the extra benefit equals the extra cost. At this point the community surplus is maximized.

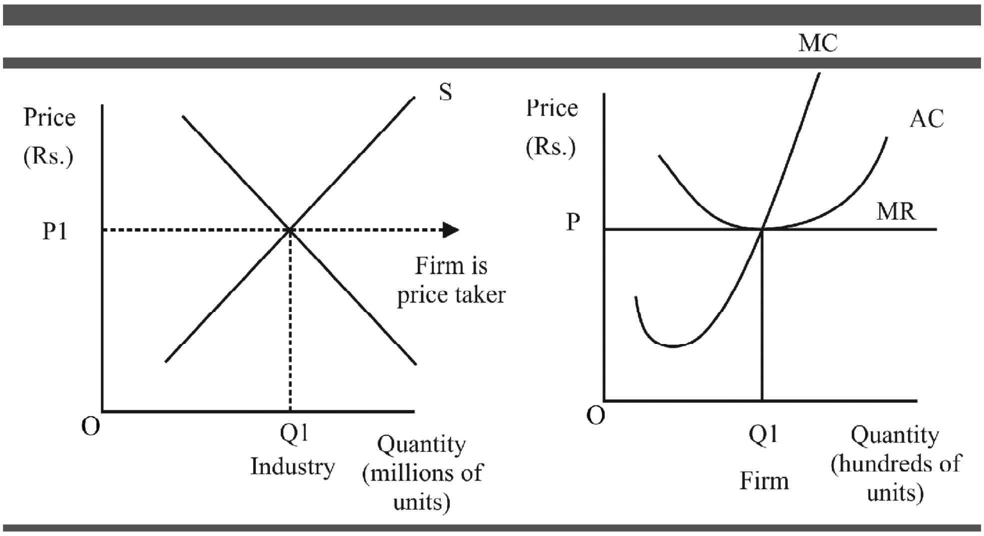


Fig 2.23: Long-run Equilibrium in a Perfectly Competitive Industry

In long-run equilibrium we have, P = AR=MR = MC = AC

In the long run firms in perfect competition are also productively efficient. Productive efficiency occurs when firms are producing at the minimum of the average cost curve; they have the lowest unit cost possible, and therefore they are not wasting resources.

To summarize, in the long run in perfect competition:

- firms earn normal profits;
- the industry is allocatively efficient (the price equals the marginal cost);
- the industry is productively efficient (firms are producing at the minimum of the average cost curve).

2.6.2 Monopoly: Basic Theory

Monopoly is a market structure characterized by a single seller of a unique product with no close substitutes. It is a market in which a single firm is the only supplier of the good. Anyone seeking to buy the good must buy from the monopoly seller. This single-seller status gives monopoly extensive market control. It is a price maker. The market demand for the good sold by a monopoly is the demand facing the monopoly. Market control means that monopoly does not equate price with marginal cost and thus does not efficiently allocate resources.

Characteristics of Monopoly

The four key characteristics of monopoly are: (1) a single firm selling all output in a market, (2) a unique product, (3) restrictions on entry into the industry, and (4) specialized information about production techniques unavailable to other potential producers.

- Single Supplier: First and foremost, a monopoly is a monopoly because it is the only seller in the market. The word monopoly actually translates as "one seller." As the only seller, a monopoly controls the supply-side of the market completely. If anyone wants to buy the good, they must buy from the monopoly.
- Unique Product: A monopoly achieves single-seller status because the good supplied is unique. There are no close substitutes available for the good produced by a monopoly.
- Barriers to Entry: A monopoly often acquires and generally maintains single seller status due to restrictions on the entry of other firms into the market. Some of the key barrier to entry are: (1) government license or franchise, (2) resource ownership, (3) patents and copyrights, (4) high start-up cost, and (5) decreasing average total cost. These restrictions might be imposed for efficiency reasons or simply for the benefit of the monopoly.

- Specialized Information: A monopoly often possesses information not available to others. This specialized information comes in the form of legally-established patents, copyrights, or trademarks.

A. Sources of monopoly power or Why does monopoly arise?

A Firm is monopoly if it is the sole seller of its product and if its product does not have close substitutes. The fundamental cause of monopoly is barriers to entry: A monopoly remains the only seller in its market because other firms cannot enter the market and compete with it. Barriers to entry, in turn, have three main sources:

- Monopoly resources: A key resource required for production is owned by a single firm.
- Government regulation: The government gives a single firm the exclusive right to produce some good or service.
- The production process: A single firm can produce output at a lower cost than can a larger number of producers.

Characteristics of Monopoly

The four key characteristics of monopoly are: (1) a single firm selling all output in a market, (2) a unique product, (3) restrictions on entry into the industry, and (4) specialized information about production techniques unavailable to other potential producers.

- Single Supplier: First and foremost, a monopoly is a monopoly because it is the only seller in the market. The word monopoly actually translates as "one seller." As the only seller, a monopoly controls the supply-side of the market completely. If anyone wants to buy the good, they must buy from the monopoly.
- Unique Product: A monopoly achieves single-seller status because the good supplied is unique. There are no close substitutes available for the good produced by a monopoly.
- Barriers to Entry: A monopoly often acquires and generally maintains single seller status due to restrictions on the entry of other firms into the market. Some of the key barriers to entry key are: (1) government license or franchise, (2) resource ownership, (3) patents and copyrights, (4) high start-up cost, and (5)

decreasing average total cost.. These restrictions might be imposed for efficiency reasons or simply for the benefit of the monopoly.

- Specialized Information: A monopoly often possesses information not available to others. This specialized information comes in the form of legally-established patents, copyrights, or trademarks.

Revenue under Variable Price Situation

For market structures like monopoly, oligopoly, and monopolistic that are price makers rather than price takers, total revenue is little different/

The vertical axis measures total revenue and the horizontal axis measures the quantity of output (Fig 2.24).

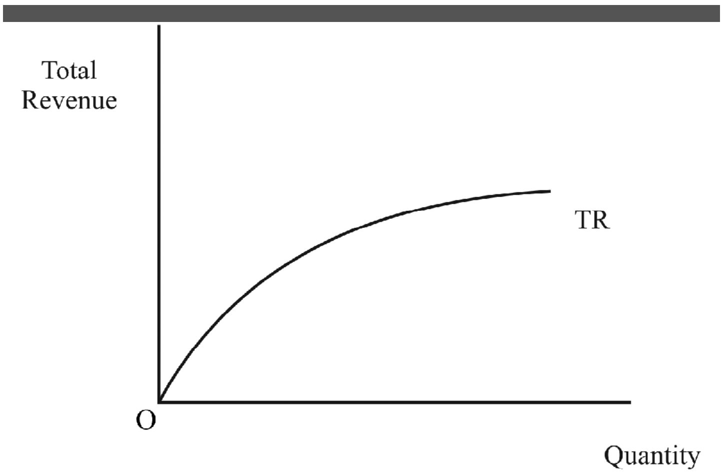


Figure 2.24: Total Revenue Curve in variable price situation under imperfect Competition

The total revenue "curve" actually is a "curve." The slope of this curve falls as more output is produced, eventually reaching a peak, then becoming negative. The changing slope of this curve is due to the changing price. Monoploist faces negatively-sloped demand curve.

Average and Marginal Revenue

Relationships among Total Revenue (TR), Average Revenue (AR) and Marginal Revenue (MR)

The relationships among TR, AR and MR are depicted in the Fig 2.25.

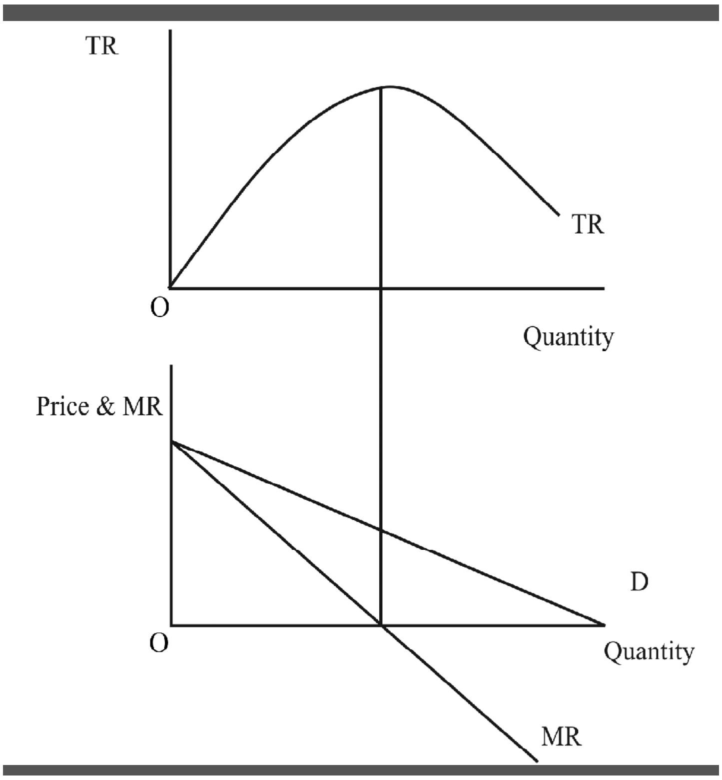


Fig 2.25 Relationships among TR, AR and MR

i) The MR will pass through the mid point of the quantity axis i.e.

ii) Slope of MR curve is twice the slope of AR

iii) When MR=0, TR is maximum. It means that when addition to total revenue is 0, TR is maximum.

iv) When MR>0, TR increases

v) When MR<0, TR increases

Relation between Marginal Revenue, Price (Average revenue) and Elasticity of Demand

Total Revenue (R) equals price (P) multiplied by quantity sold (Q)

$$R_n = PQ$$

The relation between Total and Marginal Revenue and Price Elasticity of Demand (Ep) can be specified:

If price changes from P_1 to P_2, the Δ in total revenue (R) is

$$\Delta R = (PQ)_2 - (PQ)_1$$

Or $$\Delta R = (P_1 + \Delta P)(Q_1 + \Delta Q) - (PQ)_1$$

where ΔQ = change in quantity, ΔP = change in price.

Rationalizing and omitting the subscript yields:

$$\Delta R = P\Delta Q + \Delta PQ + \Delta P\Delta Q$$

Since only small adjustments in price being considered, the last term in the above equation can be ignored to give the expression.

$$\Delta R = P\Delta Q + \Delta PQ$$

Dividing by ΔQ on both sides

$$\frac{\Delta R}{\Delta Q} = P + \frac{\Delta P.Q}{\Delta Q}$$

Or, $$MR = P + \frac{\Delta P.Q}{\Delta Q} \qquad \left[MR = \frac{\Delta R}{\Delta Q}\right]$$

Dividing by P yields

$$\frac{MR}{P} = 1 + \frac{\Delta P}{\Delta Q} \cdot \frac{Q}{P}$$

Now, $$e = -\frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

So, $$-\frac{1}{e} = \frac{\Delta P}{\Delta Q} \cdot \frac{Q}{P}$$

Therefore $MR = AR\left(1 - \frac{1}{e}\right)$ where P is always equal to AR

The formula for $MR = P\left(1-\frac{1}{e}\right)$ implies that if a demand curve is linear, the corresponding MR curve has the same vertical intercept but twice the slope. To derive this implication, consider the linear demand curve P = $a - bQ$, where a (the vertical intercept) and b (the absolute value of the slope) are positive constants. The reciprocal of the slope of this demand curve is $\frac{\Delta Q}{\Delta P} = -\frac{1}{b}$ and the own-price elasticity of demand is $e = \frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q} = -\frac{1}{b}\cdot\frac{a-bQ}{Q}$. Substituting the right side of the last equation for e in the formula for MR, we find that MR = $P\left(1-\frac{bQ}{a-Q}\right) = P\left(\frac{a-bQ-bQ}{a-bQ}\right)$ = $a - 2bQ$. Comparing the demand curve and the MR curve, we see that their vertical intercepts are the identical but the slope of the MR curve (–2b) is twice that of the demand curve.

MR > 0 only if demand is elastic so that e > 1. Similarly, if demand is inelastic (i.e., if we know that e < 1), the MR < 0. And, of course, MR = 0 when e = 1. For a linear demand curve, therefore, marginal revenue is positive above the midpoint, negative below the midpoint, and equal to zero at the midpoint.

Sources of monopoly power

Sources of monopoly power include legal barriers, economies of scales and control over important inputs.

- Legal barriers: For a firm to maintain its monopoly power there must be barriers of entry of new firms. In the case of legal barriers, the government might franchise only one firm to operate an industry, as is the case for postal services in most countries. The government can also provide licensing designed to ensure certain level of quality and competence. Workers in many trade industries most obtain government licensing for instance.

- Economies of scale: The situation in which one large firm can provide the output of the market at a lower cost than two or more smaller firms is called a natural monopoly. With a natural monopoly it is a more efficient to have one firm to produce the good. The reason for the cost advantage is economies of scale, that is, ATC falls as output expands throughout the relevant output range, as seen in Figure 1.1 below.

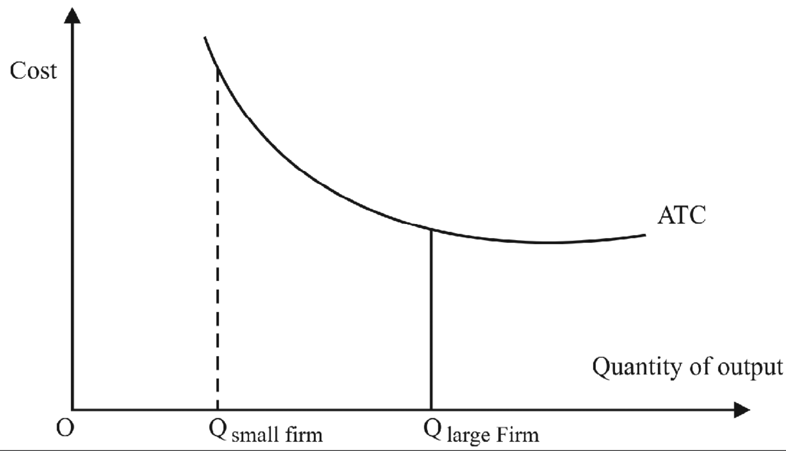


Fig 2.26: Natural Monopoly

Public utilities, like water, gas and electricity are often given exclusive monopoly rights because the government believes that they are natural monopolies.

- Complete Control over an important input : A single firm may control the entire supply of a basic input that is required to manufacture a given output. DeBeers Diamond Company of South Africa has monopoly power because it controls roughly 75 per cent of world's output of the Diamond.

Patents: A firm may acquire a monopoly power over the production of a good by having patents on the product.

B. Lerner's Degree of Monopoly Power

Economists often use the Lerner index (named after Michigan State University Professor Abba Lerner) to measure monopoly or market power. The index (L) is quite simple:

$$L = (P - MC) / P$$

which is the difference between the price (P)and marginal cost (MC) divided by Price and has a value between 0 and 1 depending upon the monopoly power of the firm in question. With regard to Perfect competition where P=MC, the Lerner Index= 0. And the larger the Index, the greater would be the degree of Monopoly power.

The Lerner index can also be expressed in terms of the reciprocal of the price elasticity of demand (e) for the firm's product. How can we get this? We know that

MR= (1-1/e) where P is price, MR is marginal revenue, e is the price elasticity facing the firm.A profit maximizing firm will set MC =MR, so MC= P (1-1/e) where MC represents marginal cost. Clearly, then MC/P=1-1/e and so 1/e=1-MC/P=(P-MC)/ P=Lerner index.

C. Profit Maximization in the Short- run

Whereas in perfect competition there are many sellers of an identical product, with **a 'pure' monopoly** there is a single seller of a product for which there is no close substitute. The firm is the industry, and, unlike a perfectly competitive firm which is a price taker, a monopolist is a price maker. This means that the demand curve is downward sloping from left to right as in **Fig 2.27**.

Since the demand curve (average revenue curve) is downward sloping, marginal revenue must be less than the average revenue. The monopoly diagram is given in **Fig**. 2.27, with marginal revenue such that it cuts the horizontal axis halfway between the origin and where the average revenue curve cuts the horizontal axis. The monopolist profit maximizes where marginal cost (MC) equals marginal revenue (MR). The price

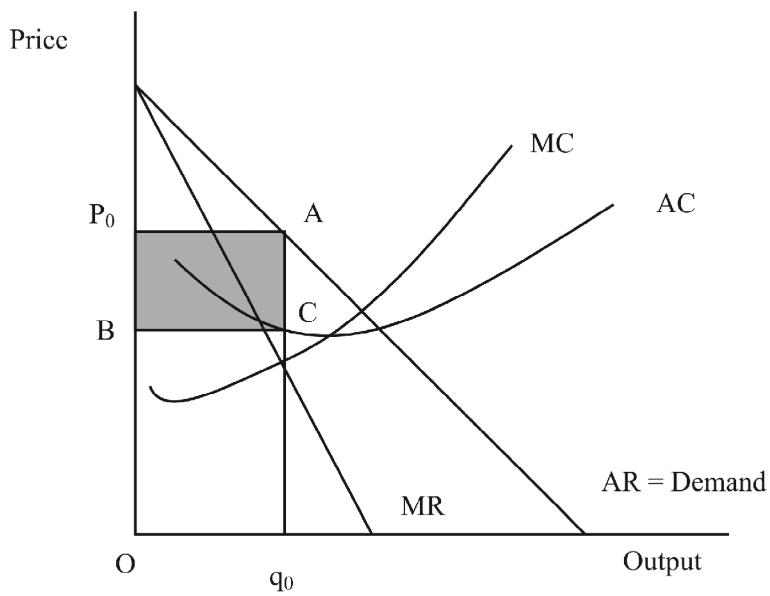


Fig. 2.27. Equilibrium of a Firm Under Monoploy

charged is P_0 and an output of q_0 is produced, and this results in the supernormal profit of the shaded area. Total revenue is equal to OP_0Aq_0 and the total cost $OBCq_0$, giving supernormal profit as stated of BP_0AC. As with a perfectly competitive firm, the monopolist must make sure that it is covering its average variable cost in the short run.

Note :

The first-order condition i.e. the equalization of MR and MC can be satisfied in each of the two cases presented in Fig. (a) and (b). The second-order condition requires that the algebraic value of the slope of the MC curve exceed that of the MR curve, i,e. the MC curve must cut the MR curve from below. This condition is satisfied at the intersection points in (a). MR = MC does not yield a point of maximum profit in (b) since the MC curve cuts the MR curve from above at it's the only point of intersection. The first-order condition can be satisfied, but the second-order condition cannot.

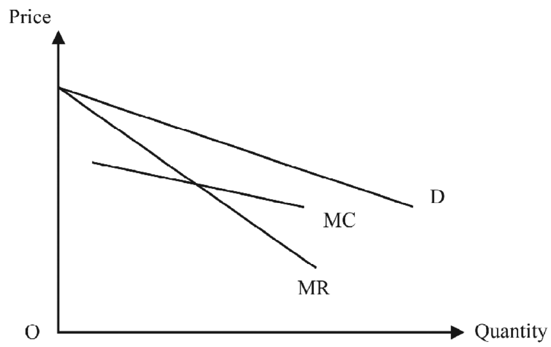


Fig: (a)

Fig: Equilibrium possibility under monopoly

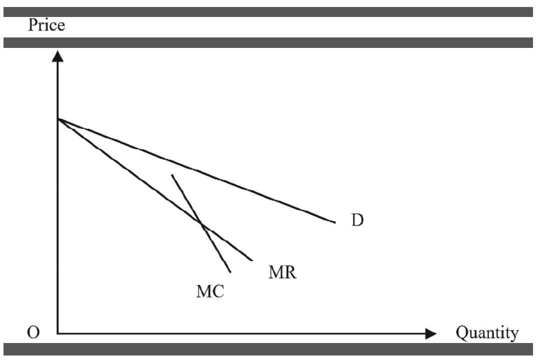


Fig: (b)

A Loss-making

A Loss-making Monopoly :

If the average cost curve is above the average revenue line it is impossible for the firm to make a profit. In the case of monopoly this is no different. At the profit maximizing output where MC = MR, the average revenue = Rs.10 and the average cost = Rs.20. The firm is making a loss per unit of Rs.10. Total losses are the loss per unit times the number of units, (Rs.10 ×Q). (2.28)

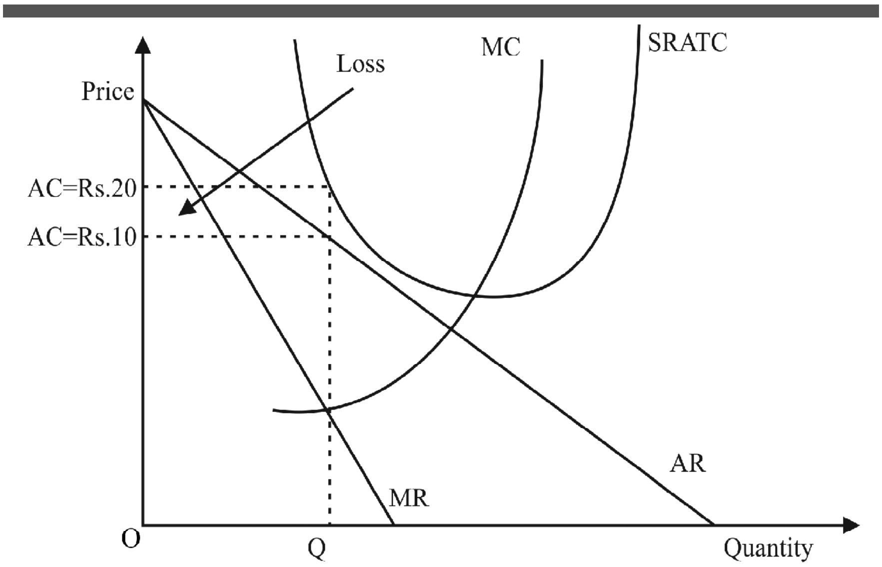


Fig 2.28. : A loss making monopoly

Some Abnormal Situations in case of Equilibrium Under Monopoly

Example: Suppose a monopolist faces a linear demand curve. Her demand function is p=100-4q. Her cost function is C=50+ 20q.

(a) Find out his profit maximizing q and p as also the amount of profit.

(b) What happens if the government forces the monopolist to follow the competitive pricing strategy? Will it shut down?(Ans: by producing a larger q=20 and selling it in lower price, p= 20, the monopolist makes a loss of Rs.50

[**Ans:** $\pi = (100q-4q^2)-(50+20q)$

Setting MR=MC, we have 100-8q= 20

Therefore , q=10, p=60, $\pi = 350$

The second-order condition is satisfied: the rate of change in MC (zero) exceeds the rate of change of MR (-8). If the monopolist is to follow the rule of the perfect competitor and set price equal to MC:

100-4q =20

Then q=20 p=20 = (—)50

Hence she will sell a larger quantity at a lower price and earn a smaller profit.]

Why is there no supply curve under Monopoly?

A monopolist in order to maximise profits does not equate price with marginal cost; instead he equates marginal revenue with marginal cost. As a result, shifts in demand causing changes in price do not trace out a unique price-output series as happens in case of a perfectly competitive firm.

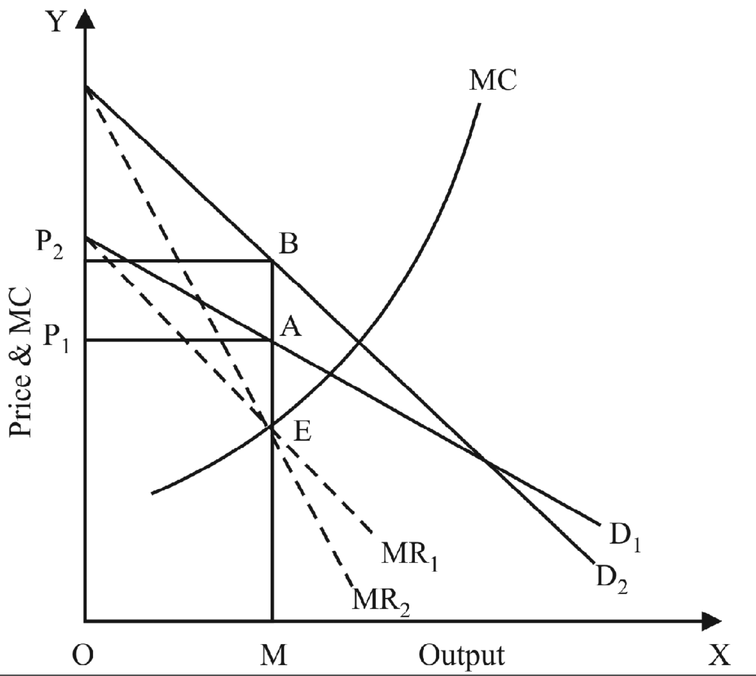


Fig 2.29: No Supply Curve under Monopoly

In fact, under monopoly shifts in demand can lead to a change in price with no change in output or a change in output with no change in price or they can lead to changes in both price and output. This renders the concept of supply curve inapplicable and irrelevant under conditions of monopoly.

Thus, there is no unique price-quantity relationship, since quantity supplied by a firm under monopoly is not determined by price but in-stead by marginal revenue, given the marginal cost curve (Fig 2.29).

What is Welfare or Deadweight loss in Monopoly:

The monopolist produces at an output where the price is greater than the MC. Because P> MC, the value to society from the last unit produced is greater than its costs. That is, the monopoly is not producing enough of the good from society's perspective. We call the area under A in the above as the welfare or deadweight loss due to monopoly.

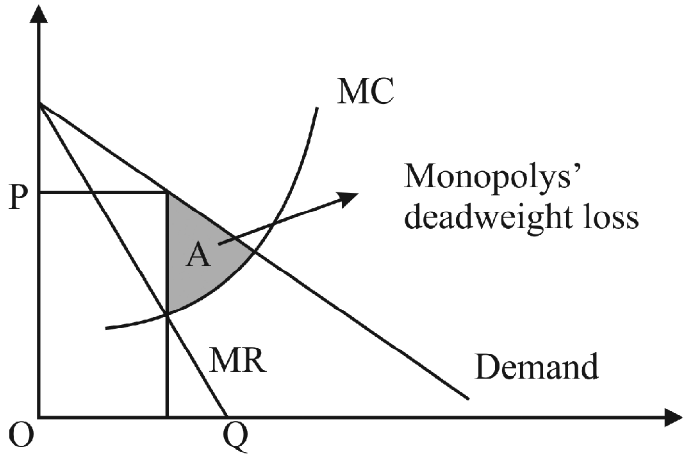


Fig 2.30: Welfare or Deadweight loss in Monopoly

D. Social Cost of Monopoly

A.E. Haberger is the first economist to explain *the social cost of monopoly*. Haberger in his article, "Monopoly and Resource Allocation", published in 1954 gave the basic arguments regarding social cost of monopoly.

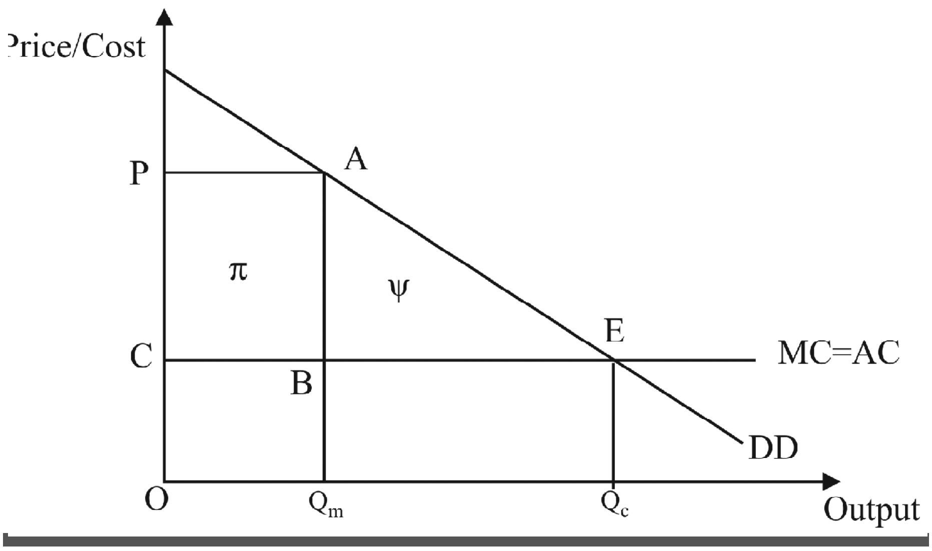


Fig 2.31 the Net Social Loss under Monopoly

Assume that long-run average costs are constant for both firms and industry and are represented by the line MC= AC. The perfectly competition output would be at Q_c where MC interests the demand curve DD. If a monopolist were substituted, she could 'maximize profits by producing Q_m at price P Her monopoly profit ð, would be represented by the rectangle ABCP. The loss of umers' surplus is measured by the trapezoid AECP. The part of the area represented by ABCP, however, has not destroyed welfare but simply a transfer of wealth from consumers to the monopolist. The net loss to society as a whole from the monopoly is given by the 'welfare triangle' ABE, denoted in **Fig 2.31** by ψ.

After making some heroic assumption, in particular that MC was constant and that price elasticity of demand was unity everywhere, Harberger estimated an welfare loss of $59 million for the US manufacturing sector in the 1920s.

Criticism: Economist like George Stigler (1956) objected that monopolist normally produced in the range where elasticity of demand is greater than one. D.R. Kamerschen (1966) reported an annual welfare loss due to monopoly in the 1951-61 period amounting to around 6 per cent of national income.

All such criticisms have obviously been of a technical nature and implicitly accept Harberger's basic methodology.

E. Price Discrimination under Monopoly: Different Degrees

Price discrimination is the practice of selling identical products at different prices at different markets. To engage in price discrimination, a firm needs to be able to (a) set prices, (b) sort buyers according to the price elasticity of demand, and (c) thwart resale. For example, the price of an air ticket may vary depending on what time of day you travel.

Analysis of Price discrimination was initiated in the mid nineteenth century by Jules Dupuit (1804-66) and other French engineers.

Different degrees of price discrimination:

1. **First degree price discrimination**: This is where the firm charges its consumer the maximum price he or she is prepared to pay for each unit. For example, stallholders in a bazaar will attempt to do this while bartering with their customers.

2. **Second degree price discrimination**: This is where the firm charges customers different price according to how much they purchase. It may charge a higher price for the first so many units, a lower price for the next so many units and so on. For example, Electricity Company in some countries charged a higher price for the first so many kilowatts.

3. **Third degree price discrimination**: This occurs where the firm divides consumers into specific groups and charges each group a different price for the same product. A.C. Pigue, an English economist in his 'The Economics of Welfare' (1950) first coined the term 'Third degree price discrimination'.

Conditions for effective price discrimination:

There are three conditions necessary for price discrimination:

i) The firm must be able to set its price. Thus price discrimination is impossible under perfect competition, where firms are price-takers.

ii) The markets must be separate. The consumers in the low-priced market must not be able to re-sale the product in the high-priced market. Markets can be separated in many ways, such as the following.

   a) Time: This means that people pay different prices at different times of day (e.g., peak and off-peak travel).

   b) Region: For example, charging different prices for the same model of car or the same beer in different parts of the world; the transport costs to buy the cars in the cheaper market and bring them back can ensure that it is not worth trying to buy in the lower-priced market.

   c) Status: For example, some firms may have customer clubs or loyalty schemes and charge different rates to members and non-members.

   d) Income: The price charged may vary according to how much you earn. Some schools offer concessions to the student who come from low-income backgrounds. However, for this to work the business must be sure that it can tell accurately what people earn, otherwise everyone will pretend to be on a low income to try to obtain a concession!

iii) The third requirement is that the price elasticity of demand is different, that is, demand is more price inelastic in one market segment than another, enabling prices to be increased in one market and reduced in others. The fact that demand conditions vary enables different prices to be charged. The higher price will be in the more price inelastic segment of the market as this will increase revenue.

First- Degree Price Discrimination

It is sometimes called perfect price discrimination where the firm sells each unit of a product separately, charging the highest price each consumer is willing to pay.

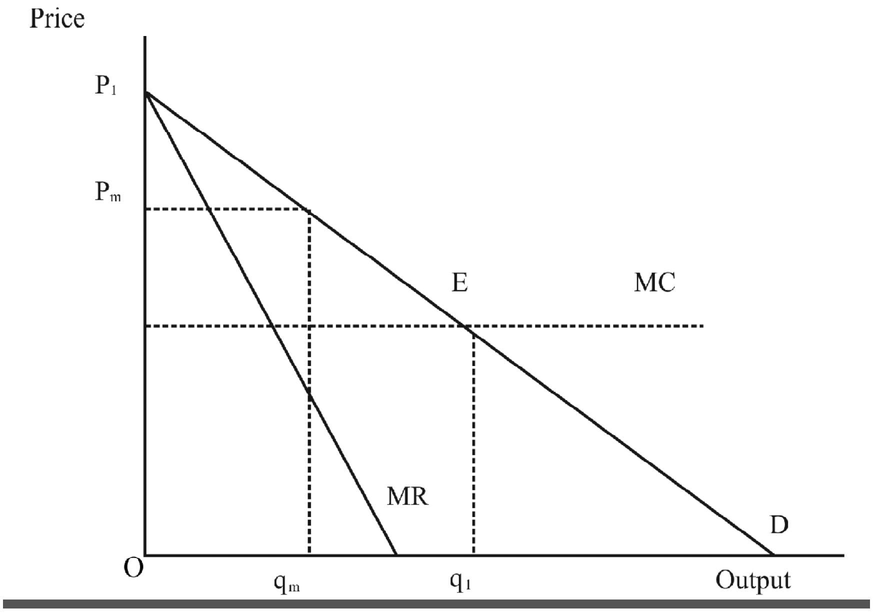


Fig 2.32. : First Degree Price Discrimination

In the first degree price discrimination the monopolist charges the consumer the highest price they are prepared to pay for each unit sold. The result is that the entire consumer's surplus is obtained by the monopolist (fig 2.32).

Second- Degree Price Discrimination

With second- degree price discrimination the monopolist charges different prices for the various blocks of the products sold. Thus a price of P_1 is charged for the first block sold, P_2 for the second block sold and P_3 for the third block sold.

Example of second- degree price discrimination is two-part tariff. A **two-part tariff** consists of an entry free and a user charge. For example, an amusement park like Nicco Park at Kolkata may charge for admission and for each ride. At a park charging Rs. 20 for entry and Rs 1 for each ride, the total cost per ride is Rs. 11 for a consumer who takes two rides and Rs. 3 for a consumer who takes ten rides (2.33).

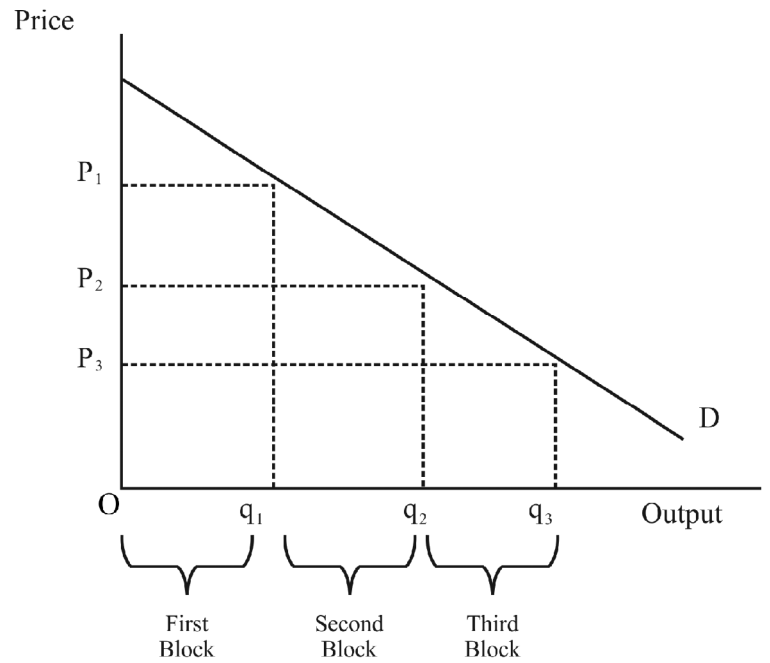


Fig 2.33 : Second Degree Price Discrimination

Third- Degree Price Discrimination

Assuming that a firm wishes to maximize profits, what discriminatory prices should it charge and how much should it produce?

Assume that the firm sells an identical product in two separate markets X and Y with demand and MR curves as showing in the diagrams.

Diagram (c) shows the MC and MR curves for the firm as a whole. This MR curve is found by adding the amounts sold in the two markets at each level of MR (in other words, the horizontal addition of the two MR curves). Thus, for example, with output of 1000 units in market X and 2000 in market Y, making 3000 in total, revenue would increase by Rs.5 if one extra unit were sold, whether in market X and Y.

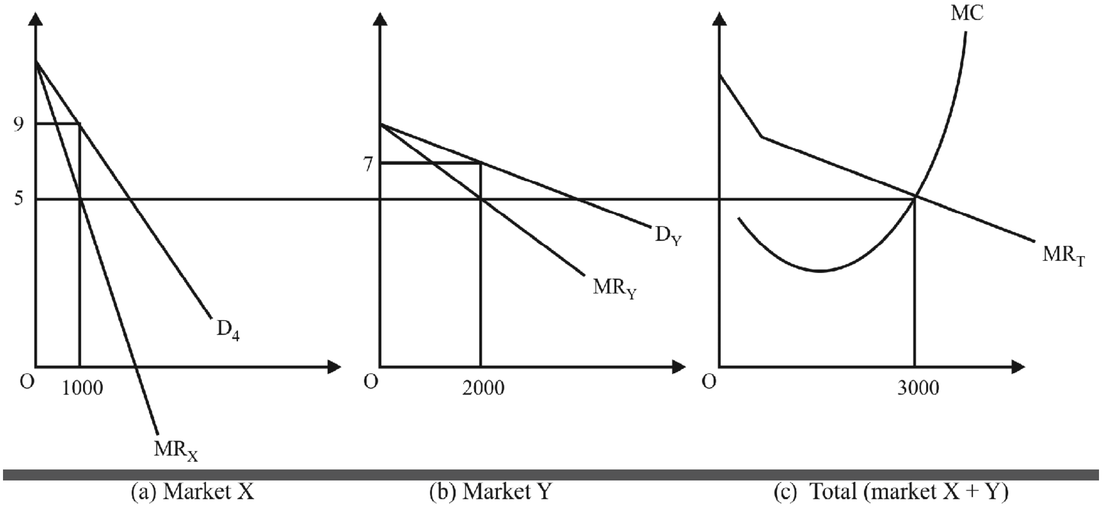


Fig2.34: Profit-maximizing output under third-degree price discrimination.

Total profit is maximized where MC = MR: i.e. at an output of 3000 units in total. This output must then be divided between the two markets so that MC is equal to MR in each market: i.e. MC = MR = Rs.5 in each market. MR must be the same in both markets, otherwise revenue could be increased by switching output to the market with the higher MR.

The profit-maximizing price in each market will be given by the relevant demand curve. Thus, in market X, 1000 units will be sold at Rs.9 each, and in market Y, 2000 units will be sold at Rs.7 each (Fig;2.34).

When price discrimination is profitable?

If there are two markets and marginal revenues in both markets are equal, then MR1 = MR2 would mean

$$P_1\left(1-\frac{1}{e_1}\right)=P_2\left(1-\frac{1}{e_2}\right)$$

Hence $$\frac{P_1}{P_2}=\left(\frac{1-\frac{1}{e_2}}{1-\frac{1}{e_1}}\right)$$

If $e_1 = e_2 = e$, then $P_1 = P_2$ and price discrimination is not possible.

If $e_1 > e_2$, then $P_1 < P_2$, hence price will be lower in Market 1 when elasticity demand is higher.

If $e_1 < e_2$, then $P_1 > P_2$

F. Peak- load Pricing

Peak- load pricing is nothing but the charging of higher prices to consumers at times of peak demand to reflect the higher costs of supplying them then. There are many examples of this in the supply of energy and passenger transport, e.g. charging less for the electricity consumed by night storage heaters and lower rail fares on days with less traffic. Industries which need this type of pricing are characterized by a demand which fluctuates over a cycle of a day or a year. The higher prices charged to pay for extra capacity cannot be too great, otherwise the consumers will shift their demand to other times which will make it increasingly difficult to raise sufficient revenue to cover the costs of peak-load special capacity. In other words,when the demand is higher at some times of the day/month than at other times, a firm may increase its profits by following the peak-load pricing strategy. i.e. charging a higher price during peak times for a commodity/ service than is charged during off-peak times. For example, the price of air ticket is higher during day time as compared to the price after 9 p.m. Similarly, the price of boat cruise ticket in Goa is higher during 7 p.m. - 9 p.m. and lower before 7 p.m. Further, gold price is higher during marriage season (usually April - June and November - January) as compared to its price in other seasons.

Peak- load pricing is thus one of the extensions of monopoly pricing. This in fact addresses the rationing of stochastic individual demands in the presence of capacity constraints. The nonlinear tariff here attempts to distribute the cost of installed capacity among consumers according to their usage, as consumers with different loads contribute differently towards the cost f providing the service. But this peak-load pricing solusion also provides the firm with incentives to reduce the size of the consumers' loads in order to minimize the cost of distributing efficiently the existing capacity among all consumers.

Figure 2.35 demonstrates the demand for electricity during the day. Demand curve D_1 represents demand at off-peak hours at night. The electricity utility company will charge a price P_1 for the off-peak hours. The costs of producing electricity increase dramatically during peak hours. Electricity generation reaches the capacity of the generating plants, causing larger quantities of electricity to be expensive to produce. For large coal-fired plants, when capacity is reached, the firm will use natural gas to generate the peak demand. To cover these higher costs, the firm will charge the higher price P_2 during peak hours. The same graph represents a large number of other goods that have peak demand at different times during a day, week, or year (ski resorts, toll roads, parking lots, etc.).

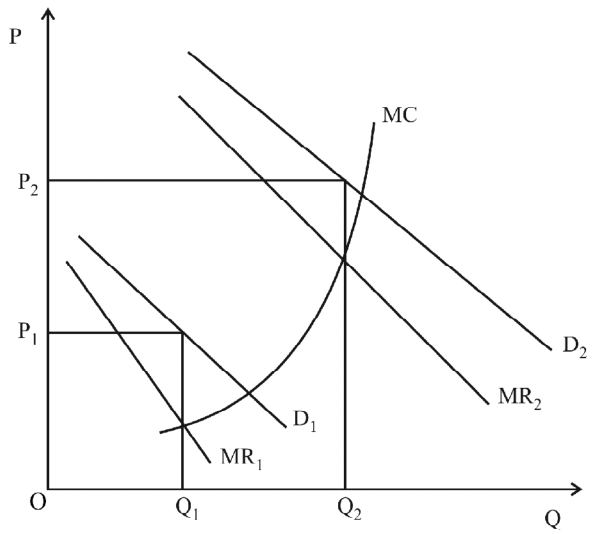


Figure 2.35 : Peak Load Pricing

Economic efficiency is greatly improved by charging higher prices during peak times. If the utility were required to charge a single price at all times, it would lose the ability to charge consumers an appropriate price during peak demand periods. Charging a higher price during peak hours provides an incentive for consumers to switch consumption to off-peak hours. This saves society resources, since costs are lower during those times.

2.7 Conclusion

In conclusion , what we can say is that we have traversed a long from the mechanics of production function, isoquants and its properties , cost function along with different market forms like perfect competition, monopoly price discrimination under monopoly and peak load pricing system.

2.8 Summary

Production : Production can be defined as the transformation of inputs, namely land, labour and capital, into goods and services in order to satisfy human wants.

Production Function : Production function is a mathematical equation showing the maximum amount of output that can be produced from any specified set of inputs given the existing of technology or state of the art.

Leontief Production Function : Q = min(aK, bL). Its interpretation is simply that Q is equal to either aK or bL, whichever of the two is smaller.

Cobb-Douglas Production Function : Q = AKaLb ,Where A is a positive constant, and a and b is a positive fraction.

CES or **ACMS production : $Y= A[\alpha K^{-\rho} + (1- \alpha) L^{-\rho}]^{-1/\rho}$**

Law of Production in the Short Run : In the short run, as a firm employs more of the variable factor, it will eventually experience *diminishing returns to the variable factor*. This is known as the law of diminishing returns or the law of variable proportions.

Isoquant : An isoquant can be defined as a curve showing the various combinations of capital and labour required to produce a given quantity of a particular product, in the most efficient way.

Properties of Isoquants : (i) Isoquant is negatively sloped, (ii) Two isoquants can never intersect or touch, (iii) isoquant is convex to the origin

Marginal rate of technical Substitution (MRTS): The rate at which one input may be traded-off against another in the production process while holding output constant.MRTS is the absolute value of the slope of an input.

Iso-cost : An **isocost line** is a **line** that represents all combinations of a firm's factors of production that have the same total cost.

Optimal employment of inputs : Input levels that minimizes cost.

Cost: The cost of producing something is the value of the cheapest bundle. There are several types of costs. *Fixed costs* are associated with *fixed inputs*. **Variable** costs are associated with *variable inputs*. The **total** cost of a specified quantity of output include both fixed and variable costs. **Average cost** is total cost divided by output.

The **marginal cost** of the additional output is the change in total cost divided by the associated change in output.

Total Fixed Cost Curve : The total fixed cost (TFC) curve is a horizontal line. Total fixed cost does not change with the quantity of output produced, thus the TFC curve is a flat, horizontal line.

Total Variable Cost Curve : The total variable cost (TVC) curve is a positively-sloped line that reflects increasing then decreasing marginal returns. The TVC curve emerges from the origin with a relatively steep slope, flattens, then becomes increasingly steeper.

Total Cost Curve : The total cost (TC) curve can be derived as the vertical summation of the TVC and TFC curves. In other words, the TC curve can be found by shifting the TVC vertically by the amount of TFC. This means that the shape of the TC curve is identical to that of the TVC. The two curves have identical slopes for each quantity of output.

Returns to Scale :A way of classifying production function that records how output responds to proportional increases in all inputs. Mathematically, if $f(tK,tL)=t^{k}f((K,L)$,,$k>0$ implies increasing returns, $k=1$ constant returns, and $k<1$ decreasing returns.

Economies of Scale : Economies of scale exist when the expansion of all inputs, especially labour and capital, result in a decrease in long-run average cost. Economies of scale usually occur for relatively small levels of production and are then overwhelmed by the diseconomies of scale for relatively large production levels.

Diseconomies of Scale : Diseconomies of scale exist when the expansion of all inputs, especially labour and capital, result in an increase in the long-run average cost. Diseconomies of scale usually occur for relatively large levels of production.

Relationship among AC & MC : For the AC to rise, MC > AC, For the AC to fall, MC < AC & for the AC to remain constant, MC = AC

Long Run average cost (LAC) curve :The LAC is the envelope of all the SRAC curves and shows the minimum per unit cost of producing each level of output. shows the minimum per unit cost of producing each level of output when any desired scale of plant can be built.

Long Run marginal cost (LMC) : LMC measures the change in long-run total cost (LTC) per unit change in output.

Monopoly profit Maximization: Like any firm, a monopoly— a single seller— maximizes its profit by setting its output so that its MR equals MC. The monopoly makes a positive profit if its AC < P. at the profit maximizing output.

Market power: Market power is the ability of a firm to charge a price above MC and earn a positive profit. Because a monopoly does not have a supply curve, the effect of a shift in the demand on a monopoly's output depends on the shapes of both its MC curve and its demand curve.

Welfare effects of monopoly: Because a monopoly's price is above its MC, too little output is produced, and society suffers a deadweight loss.

Price discrimination: The practice of charging different prices to different customers despite the cost of production being the same.

Peak load pricing: Charging a high price during demand peaks, and a lower price during off-peak time periods.

Two part tariff: A pricing system in which a price consists of two parts: one part pays for fixed costs, other for variable costs. A major example of this is the charge of a telephone rental to cover fixed costs and the separate charging for calls to pay for variable costs. It is a form of price discrimination.

2.9 Exercises

A. Short-answer Type Questions

1. What is production function? Mention various types of production function
2. What is Leontief production function?
3. What is Cobb-Douglas production function?
4. What is CES production function?
5. What is homothetic production function ?
6. What is shut –down point and what is break even point
7. A firm uses 10 units of L and 20 units of K to produce 10 units output. The $MP_L = 0.5$. If there are constant returns to scale, then show that MP_K must be 0.25.

[Ans: For constant returns to scale, the production function is : Q= MPL .L + MPK.K or 10= 0.4 .10 + MPK .20 Therefore MPK= 0.25.]

8. What is the usefulness of the ACMS production function?

   [Ans: This function is proved to be extremely useful in empirical studies in which the investigator prefers to let the data determine elasticity of substitution rather specifying it to take some particular value *a priory]*

9. In the ACMS production : $Y= A[\alpha K^{-\rho}+(1-\alpha) L^{-\rho}]^{-1/\rho}$, what the symbols A, α and ρ stand for?

   [Ans: In the ACMS production function, A is an efficiency parameter in the sense that it merely shifts the production function;, α is a distribution parameter that permits the relative importance of K and L to vary; and ρ is a substitution parameter because it can be shown that $\sigma = \dfrac{1}{1+\rho}$. The CES or the ACMS production function therefore includes σ=1, σ=0, and σ= -1 and $\sigma = \infty$]

10. What is monopoly? What are the characteristics of monopoly?

11. Why there is no supply curve under monopoly?

12. What is two part tariff? Give example.

13. What is Peak –load pricing? How does it operate?

B. Medium-answer Type Questions

1. What is homogenous production function? State its properties.

2. State the properties of Cobb-Douglas production.

3. Suppose a monopolist faces a linear demand curve. Her demand function is p=100-4q.

   Her cost function is C=50+ 20q.

   (a) Find out his profit maximizing q and p as also the amount of profit. (Ans: q=10, p=60, π = 350)

   (b) What happens if the government forces the monopolist to follow the competitive pricing strategy? Will it shut down?]

4. Prove that a monopolist will never, in equilibrium, produce on the inelastic part of the demand curve.

5. Explain why the notion of supply curve is not very meaningful except under competitive conditions.

6. Prove that if the demand and AC functions of an industry are linear, the monopoly (maximum profit) output will exactly half of the competitive (zero profit) output. Show this either geometrically or algebraically.

7. Prove that a monopolist who has a zero MC will set the price at a level where elasticity of demand is unity.

8. Why will not a monopolist operate at a point where the elasticity of demand is unitary, if it wants to maximize profits? Is it that for a monopolist to operate at a point where price elasticity of demand is unitary, MR must be zero, and thus, MC would need to be zero, since profit maximizing condition is MR= MC. So it is not likely that monopolists will operate where price elasticity of demand is unitary. Do you think that the argument given is true?

9. What is Peak-load pricing ? How is Peak-load pricing a form of price discrimination?

C. Long –answer Type Questions

1. What does the Cobb-Douglas Production Function tell? State the properties of Cobb-Douglas production function

2. Show that for the CES production function,

   $Y= A[\alpha K^{-\rho}+(1-\alpha) L^{-\rho}]^{-1/\rho}$

   (i) $MP_k = \frac{\alpha}{A\rho} \cdot \left(\frac{Y}{k}\right)^{1+\rho}$ and $MP_L = \frac{(1-\alpha)}{A^{\rho}} \cdot \left(\frac{Y}{k}\right)^{1+\rho}$

   (ii) $MRTS = \frac{(1-\alpha)}{\alpha}\left(\frac{K}{L}\right)^{1+\rho}$

   (iii) Determine the output elasticities for K and L. Show that their sums equal to 1.

[Ans Hints: the elasticity of output w.r.t Labour(L) is: e= MP_L/AP_L

And the elasticity of output w/r.t Labour(K) is: e= MK_L/AP_K

3. State the relationship between AVC, MC, and AC
4. State the relationship between SAC, and LAC
5. Let us suppose that TC= 100+60Q- $10q^2$, when Q is the quantity of output, Find (i)

   AFC;(ii) AVC and (iii) AC if Q=5 units. Why is the Short run MC curve is U-shaped?
6. Write short notes on the following:
   (a) Measure of monopoly power.
   (b) Social Cost of monopoly
   (c) Deadweight loss under monopoly
7. Do you agree with following? Give reasons.
   (a) There is no supply curve under monopoly
   (b) Under price discrimination price is lower in the market with more elastic demand.
8. When is price discrimination possible and profitable? Show that price discrimination will generally in a higher output than in single monopoly.
9. Suppose that the total demand for a product is divided into two markets with the demand functions $Q_1=12-P_1$ and $Q_2=18-P_2$ and the MC of the output as a whole is Rs.4. Find the prices charged and quantities sold in each of the two markets, assuming a monopolist producer.

2.10 References

Clemhout, Simone (1964). The class of homothetic isoquant production functions. Paper presented to the *Econometric Society*, Chicago.

Feldstein,M.(1972). Equity and efficiency in public sector pricing: the optimal two part tariff, *Quarterly Journal of Economics* 86:175-87.

Heathfield, D. F. (1971). The Cobb-Douglas Function. In *Production Functions* (pp. 29-44). Palgrave, London.

Heathfield, D. F. (1971). *Production and Functions*. Macmillan International Higher Education.

Heathfield, D. F., & Wibe, S. (2016). *An introduction to cost and production functions*. Macmillan International Higher Education.

Sankhayan, P.L. (1988). *Introduction to the Economics of Agricultural production function.* Prentice- Hall of India, New Delhi.

Shephard, R. (1953). *Cost and Production Functions.* Princeton: Princeton University Press.

Stigler, George J. (1966): *The Theory of Price*, New York: Macmillan.

Williamson, O.E. (1966). Peak –load Pricing and optimal capacity under indivisibility constraints, *American Economic Review* 56: 810-37.

Unit 3 ❑ Alternative Theories of the Firm

Structure

3.1 Objectives

After going through this unit you will be able

- To understand Baumol's sales maximizing model;
- Williamson's model of managerial discretion;
- To have an idea about Marris model of managerial enterprise;
- to get a knowledge about full cost pricing rule; and
- Delve deep onto the Behavioural theory of the firm.

3.2 Introduction

Up to this point we have assumed that the firm is attempting to maximize profits. Wehave implicitly taken the point of view that only market considerations determine the decisions of the firm. For example, we have assumed that the internal organizational structure of the firm have no effect on the decisions that firms ma.kes. Now we shall look at the firm from a different point of viewpoint. Only the profit maximizing endeavour by the firm has become unimportant in present day complex world. In consonant with the present situation, many alternative theories of firms have sprung over time. In this unit we have presented five models which attempt to develop new approaches to the theory of the firm: Baumol's Sales Maximization model, Williamson's model of managerial discretion, Marris model of managerial enterprise, Full cost pricing rule ,and Behavioural theory of the firm

3.3 Baumol's Sales Maximization Model

In the standard analysis of a firm, the objective of profit maximization is usually assumed. However, when the firm in question is a corporation in which ownership and management are separate, it would be very rational for the management to pursue the alternative goal of maximizing the sales (revenue). Moreover, increases in sales revenue are often taken to be a sign of managerial success. Here it is true that the remuneration of the management depends directly on this particular performance index. Thus, sales maximization appears to be a plausible alternative objective in the corporate set up provided that the profit level does not fall below a certain prescribed minimum, say, π_0, which is below the maximum profit associated with the MR=MC condition.

If so, the problem of the management is to maximize R= R(Q), subject to $\pi = R(Q) - C(Q) \geq \pi_0$.

As long as R(Q) is differentiable and concave and C(Q) is differentiable and conves—which implies that the constraint function C(Q)-R(Q) is also differentiable and convex—the Kuhn-Tucker suuficiency theorem can be applied. The case of a concave R(Q) function, i.e,. TR curve and a convex C(Q) function is illustrated in the Fig below.

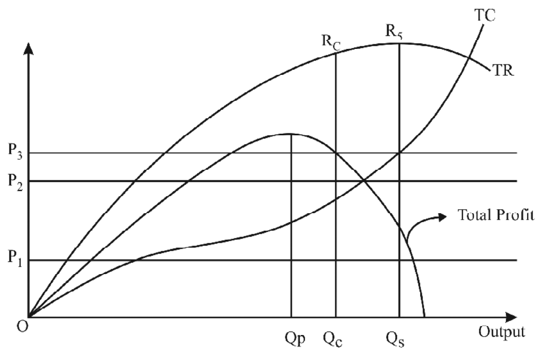


Fig 3.1: Sales maximizing model

For this problem, the Lagrangian function is $Z= R(Q)+ \lambda\ [(R(Q)- C(Q) - \pi_0]$

The Kuhn-Tucker conditions are

$$dZ/\ dQ = R'(Q) + \lambda\ [\ (R'(Q) - C'(Q)\ ] \leq \pi_0$$

$$dZ/\ d\lambda = R(Q) - C(Q) - \pi_0 \geq 0$$

plus the non-negativity and complementary-slackness conditions.

Solving, we can then obtain the constrained sales- maximizing output rule

$$R'(Q)= \lambda/(1+ \lambda)\ C'(Q)\ldots(a)$$

With a positive λ, the sales –maximization output rule (a) above indicates that

$$R'(Q) < C'(Q) \quad [\text{ since } \lambda/(1+ \lambda) <1\ ]$$

And this will generally yield a higher output level than the profit-maximizing rule MR=MC.

From the Fig 3.1, it appears that two types of equilibrium appear to be possible: one in which profit constraint does not provide an effective barrier to sales maximization, and one in which it does. This is illustrated in Fig which shows the firm's total revenue, cost, and profit curves as indicated. The profit and sales maximizing outputs, are, respectively, OQ_p and OQ_s. Now if, for example, the minimum required profit level is OP_1, then the sales-maximizing output OQs will provide plenty of profit, and that is the amount it will pay the sales maximizer to produce. His selling price will be set at QsRs/ OQs. But if the producer's required level of profit is OP_2, output OQs, which yield only profit QsPs, clearly will not do. Instead his output will be reduced to level OQ_c, which is just compatible with his profit constraint.

The profit maximizing output, OQp, will usually be smaller than the one which yields either type of sales maximization., OQs This can be proved with the aid of the rule that at the point of maximum profit MC must be equal MR. For MC is a positive number, hence MR revenue is also positive when profits are at a maximum, i.e., a further increase in output will increase total sales (revenue) Therefore, if at the point of maximum profit, the firm earns more profit than the required minimum; it will pay the sales maximizer to lower his price and increase his physical output.

3.4 Williamson's Model of Managerial Discretion

Williamson has developed a model of rational manegerial business behavour which focuses on the self- interest seeking behavior of corporate managers. The separation of ownership and management functions permits the managers of a large firm to pursue their own self-interest, subject only to their being able to maintain effective control over the firm. We can postulate here a utility function which incorporate management's goals in which the management is interested.

In this vein, O. Williamson presented the utility maximization model. In Williamson's formal model, the utility function that managers are trying to maximize may be represented as a function of three variables: (1) dollar expenditures on "staff," S; (2) management slack absorbed as cost, M; and (3) discretionary investment spending, I_D. Letting U denote the utility function, we have

$$U = U(S, M, I_D)$$

The managers attempt to maximize the value of the above utility function subject to three constraints. The first is the minimum required profits constraint, Equation.

$$\pi_R \geq \pi_O + T$$

where T denotes the total taxes of the firm, π_R is the reported profit and *Minimum required profits (after taxes),* π_O, are-the lowest level of profits consistent with the managers' retaining effective control over the firm.

An equivalent form for expressing this constraint is

$$\pi_R \geq \frac{\pi O}{1-T}$$

I_D can be treated as to be nonnegative. Two additional constraints must be imposed to insure that S and M are also nonnegative. By adopting the reasonable assumption that U exhibits diminishing marginal utility in all its components and that, as any of its components approaches zero, the marginal utility of that component becomes unbounded, then these non-negativity constraints will be automatically satisfied whenever U is maximized.

Managers derive psychic satisfaction and prestige from the number of staff under their control and from luxury offices, company cars, bonuses they get, as also the reimbursements of various types of expenses (such as entertainment, medical and travelling expenses). Managers maximise their utility subject to a minimum profit constraint.

Here we consider a simple version of the model where the utility function of the managers depends on profits and the size of the staff under their control. The indifference curves are shown in Fig. 3.2. Profits increase with an increase in the number of staff members but after a point profits fall. The profit-maximizing model gives S_0 as the optimal level of staff The utility-maximizing model results in the staff of S_1, If the minimum profit constraint is shown in Fig. 3.2, the utility-maximizing model results in a staff of S_2. The final solution will be higher level of staff and lower profits than under the profit-maximizing model.

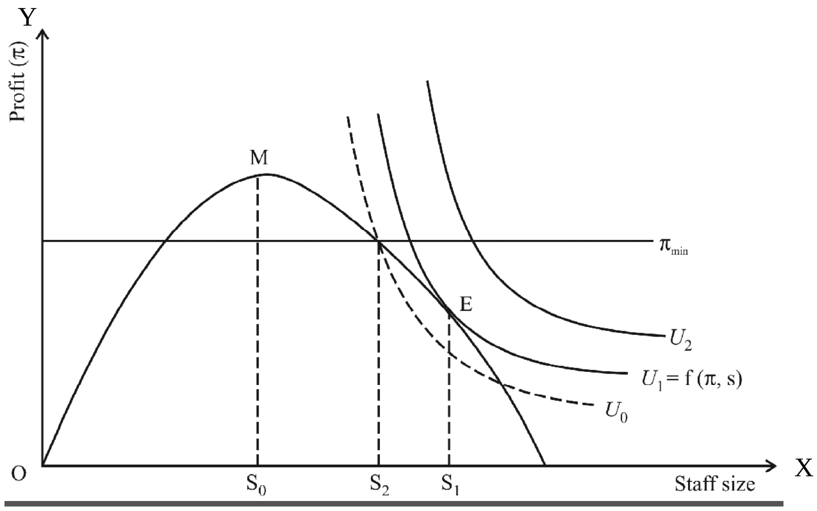


Fig 3.2 Utility-maximization Model (Utility as a function of profits and Staff Size)

Effect of Tax : Changes in the corporate tax rate produce distinctly different responses in all three models. When the corporate tax rate increases, a revenue-maximizing firm reduces its output and staff expenditures; a utility-maximizing firm increases its output and staff expenditures; a profit-maximizing firm 'does not alter its output and staff expenditures.

Rationale for the Managers' Utility Function

Williamson has developed a model of business behavior which focuses on 'the self-interest-seeking behavior of corporate managers. This emphasis is reasonable once we acknowledge that the modern corporate enterprise is a complex organization far different from the traditional economic notion of a single entrepreneur running his own small firm.

One of the striking attributes of the typical large firm in the developed economy is the separation of the ownership and the management functions which has become increasingly prevalent. The owners of the firm, i.e., the stockholders, generally have little interest in, and even less direct knowledge of', the day-to-day operations of the firm. It is the owners, however, who have contributed the capital with which the firm operates and who receive the dividends which are paid by the, firm. The actual power of the stockholders to influence the firm's plans and operations resides in the board of directors, who are elected by stockholder vote

The top management of the firm is appointed by, and responsible to, the board of directors. In many firms, however, top management is represented by membership on the board of directors and may play a dominant role on the board. Even where the board is predominantly made up of outside directors, top management usually possesses a great deal of freedom of action if the results of the firm's operations are satisfactory.

This separation of ownership and management functions permits the managers of a firm to pursue their own-self-interest, subject only to their being able to maintain effective control over the firm. In particular, if profits at any time are at an acceptable level, if the firm shows a reasonable rate of growth over time, and if sufficient dividends are paid to keep the stockholders happy, then the managers are fairly certain of retaining their power.

What might the management group's self-interest depend upon? As is the case in the traditional theory of household behavior, we can postulate a utility function which incorporates those goals in which management is interested. We can regard the managers' utility as dependent primarily upon: (i) the salaries (and other forms of monetary compensation, such as bonuses, stock options, etc.) which the managers receive from the firm, (ii) The number and quality of staff personnel who report to the managers,(iii) the extent to which the managers are able to direct the investment of the firm's resources, and (iv) The type and amount of perquisites (such as expense accounts, lavishly furnished offices, etc.) which the managers receive from the firm and which are beyond the amount strictly necessary for the firm's operations. In Fig.3.2 we take only profit and staff size for simplicity.

There can be little disagreement that salaries are a major factor affecting the well being of managers" since monetary compensation from their jobs provides them the means for financing their private life expenditures. Money alone, however, is not the entire reward which a manager obtains from his job. The staff personnel commanded by managers are-important both as a mark of status and as a measure of power.

The Formal Model

In order to formalize his model, Williamson introduces four different types of profits: maximum profits, actual profits, reported profits, and minimum required profits. The concept of *maximum profits* ð*, refers to the profits which the firm would obtain if it behaved as a profit-maximizing agent. ð* is thus the maximum value of the criterion function in traditional theories. *Actual profits*, $ð_A$, differ from maximum profits by the amount of excess expenditures for staff. Reported profits, $ð_R$, are less than actual profits by the amount of management slack(M) which is absorbed as a cost, i.e., $ð_R = ð_A - M$. *Reported profits* are the same as taxable profits; they are used to pay taxes and dividends and to finance some investment.

Minimum required profits (after taxes), $ð_O$, are-the lowest level of profits consistent with the managers' retaining effective control over the firm. This minimum level must be high enough to enable the firm to pay normal dividends to the stockholders (and perhaps to allow for gradual growth in dividends) and to provide for any economically necessary investments (which are not included in the discretionary investment spending). If there exists an expectation that profits will generally grow over time (with due allowance, of course, for cyclical fluctuations), the minimum required profit level may increase from year to year. Letting t denote the tax rate on (reported) corporate profits and letting T denote the total taxes of the firm, we have $T = t\pi_R$

In sum, Williamson's managerial discretion model assumes that the managers of the firm are trying to maximize a utility function which depends upon three components: (1) the level of staff expenditures; (2) the amount of management slack absorbed as cost (i.e., the amount of managerial perquisites); (3) the amount of discretionary spending available- for investment.

In the ultimate analysis we find that a utility-maximizing firm has higher staff expenditure and more management slack than a profit maximizing firm.

The implications of Williamson's model appear to be plausible on the basis of casual empiricism. These implications include predictions that expenditures on both staff and managerial perquisites will increase as the environment becomes favorable (e.g., as demand increases or as competition in the market weakens), and that spending on both will decrease when the environment becomes more hostile. There are also predictions about how the firm will respond to changes in corporate tax rates and to changes in fixed costs.

Since in many respects the predictions of Williamson's model differ from those of the traditional profit-maximizing model, empirical tests can be devised which should discriminate between these two models. Williamson has presented some empirical evidence based both on field studies and statistical analyses which tends to support his model of management behavior. This empirical evidence does' not constitute a complete refutation of profit-maximizing behavior by the firms, but it at least suggests that in certain types of markets, namely where competitive forces are weak, Williamson's model may have more empirical relevance than the traditional profit-maximizing models.

Baumol's revenue-maximization model assumes that a firm attempts to maximize its total revenues subject to a constraint that it earn some specified minimum amount of profits. The implications of Baumol's model differ in many respects from the implications of both Williamson's model and the traditional profit-maximization model. A revenue-maximizing firm produces at an output where its marginal revenue is less than its marginal cost; both a utility-maximizing and a profit-maximizing firm equate marginal revenue with marginal cost. A profit-maximizing firm equates the marginal revenue generated by the last dollar spent on staff to the marginal cost of this last dollar; both a revenue-maximizing and a utility-maximizing firm carry staff expenditures beyond this point.

3.5 Marris Model of Managerial Enterprise

The growth maximization model, which is inherently managerial theory of the firm, developed by Marris and Penrose is essentially concerned with the time path of expansion of the firm. Managers are supposed to satisfy instincts of power dominance and prestige (and possibly higher salary) by pursuing growth as an objective. They also take into account the valuation ratio, which is the ratio of the stock market value of the firm to its accounting or book value. The ratio is usually quoted in any type of takeover bid. In Fig. 3.3 we show the relationship between valuation ratio and the growth rate by the *valuation curve*. The valuation curve takes into account the relationship between growth and profitability and the present value of shareholders' dividends and capital gains. After a certain point the valuation ratio declines. If it is substantially below 1, then firm faces takeover threat. The indifference curves drawn in Fig 3.3 show the managers' preference for valuation and growth. The managers derive the maximum satisfaction (at point E) at the growth rate g_1 with the corresponding valuation ratio V_1. The growth rate that maximizes shareholders' equity (which is also the profit-maximizing point) is at the growth rate g_0 with the corresponding valuation ratio V_0.

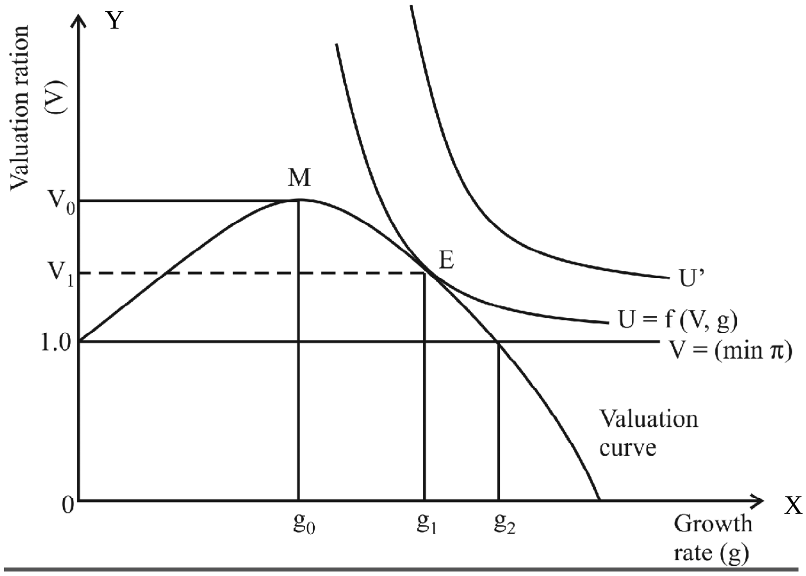


Fig 3.3: Growth Maximization Model

Comparison with Profit Maximisation Model

The growth-maximization model results in a higher growth rate and lower valuation ratio than the profit maximization model. Here we see that the maximum growth rate the mangers can pursue is g_2 corresponding to a valuation ratio of 1. Any higher growth rate will lead to a takeover threat.

3.6 Full cost pricing rule

Many firms under imperfect market structure follow a *mark-up or full-cost or cost-plus pricing strategy.* As per this pricing strategy, mark-up on cost (m) = (P-AC) / AC, where P is the price per unit of output and AC is the fully allocated average cost or the cost per unit of output (i.e. average variable cost at normal output plus allocated overhead). From the above formula, we can solve for P= AC + m(AC) = AC(1+m).

By using the marginal pricing strategy, we obtain P = $[e_p/(1+e_p)]$ MC. Similarly, by using the cost-plus pricing strategy, we obtain P = AC(1 + m). Thus, we will obtain the optimal mark-up (m) by solving these two equations. As P $[e_p/1+e_p)]$ MC and P=AC

(1+m). we can write AC(1+m) = MC [e_p/(e_p +1)]. We know that the behaviour of each profit maximizing firm is to produce the output at the lowest average cost (AC). We also know that marginal cost (MC) curve always intersects AC curve at its minimum point from below, where MC = AC. Thus, we can write AC(1+m) = MC e_p/(e_p +1) = as C(1+m) C(e_p/(e_p +1), where C represents per unit cost of production. The optimal mark-up (m), therefore, is m [e_p/(e_p+1)]-1, where e_p is the price elasticity of demand for the product which usually carries a negative sign.

From this pricing rule, the manager must note two important things. First, the greater is the absolute value of price elasticity of demand for the firm's product, the lower is the profit maximizing mark-up (ie. optimal mark-up) and vice versa. If the demand for a firm's product is more elastic (ie many substitutes are available for the firm's product), the manager of the firm sell such a product by charging relatively low mark-up and vice versa if the demand for the product is less elastic. Secondly, the higher is the cost marginal cost of the firm, the higher is the profit-maximizing price and vice versa. In other words, if the marginal cost of producing a product by a firm is higher, it will charge a higher price for its product as compared to firms with lower marginal cost, ceteris paribus.

3.7 Behavioural theory of the firm

(I) Motivation of the Behavioral Theory

Periodically in the history of the theory of the firm in economics, there have been attacks on the assumption of profit maximization. In particular, as one looks closely at the behavior of actual firms, the justification for the assumption of profit maximization seems to weaken. When one adds uncertainty to the firm's decision-making process, even defining the meaning of profit maximization becomes difficult to do in an empirically meaningful way: The behavioral theory of the firm takes the position that arguments over motivation are somewhat fruitless. The critical issue is not whether one assumes profit maximizing instead of satisfying behavior. Instead, it is fruitful to develop an understanding of the process of decision making within the firm.

The behavioral theory is viewed as supplementing the conventional theory of the firm. The traditional theory is essentially one in which certain broad questions are asked. Specifically, the conventional theory of the firm is designed to explain the way in which the price system functions as a mechanism for allocating resources among markets; relatively little is said about resource allocation within the firm. For the purposes of the classical theory, the profit maximization assumption may be perfectly adequate. It is clear, however, that as one asks a different set of questions, specifically questions designed to uncover the way in which resources are allocated within the firm, the profit maximization assumption is neither necessary nor sufficient for answering these questions. Therefore the behavioral theory of the firm should be viewed as focusing on a different set of questions, questions concerning the internal decision-making structure of the firm. Thus, with this theory we are interested in answering such questions as.

1. How does the allocation of resources within the firm's budget relate to the organizational goals?
2. How do objectives change over time?
3. What happens to information as it flows through the organization?
4. Are there biases in the information?
5. How do these biases affect the decisions that are finally made?
6. What is the relationship between decisions made by management and tile final form of the decision as it is implemented by the organization?

In general the behavioral theory is most applicable to those firms whose decisions are not completely determined by the market. These firms have some freedom to develop decision strategies or rules that become part of the decision-making system-within the firm.

(II). Key Concepts or the Behavioral Theory

A. ORGANIZATIONAL GOALS

The phrase, "organizational goals," is slightly misleading, since an organization as such cannot have goals. Only the individuals within the organization can have goals. When we speak of organizational goals, however, we mean essentially that there is agreement among some-group responsible for the direction of the organization on the nature of the goals. In particular, this group (which we shall call a coalition within the organization) is an interacting group, and the goals will be modified by discussions and pressures within the group. Thus the decision on the final set of goals of the organization is in some sense a political decision. The coalition within the organization for a business firm may include managers, workers,' stock- holders, customers, and so on. In other words, all the individuals who have some stake in that particular organization may in one way or another affect the goals of the organization.

The concept of a coalition assumes a different type of firm than we have been dealing with in the preceding chapters of 'this book. Hitherto, the firm was implicitly regarded as controlled by an entrepreneur, and the goals of the entrepreneur were the goals of the organization. He purchased conformity to these goals by payments in the forms of wages to workers, interest to capital sources, and profits (when they existed) to himself.

To understand organizational goals it must be realized, first of all, that all resolutions of goals within the coalition are not made by money. Rather, many side-payments to members in an organization are made in the form of policy commitments. For example, in order to get the Vice-President of marketing to stay within the organization, it may be necessary to commit resources to research on new products. It may well be that some of the policy demands are inconsistent with the side-payments that are made. Thus, every organization is continually undergoing the test of new demands, the test to see how these new demands conform to existing policy and, in general, pushing the policy toward new dimensions. In some sense, therefore, the goals of the organization are never completely consistent at any particular point in time.

The second point that must be understood- about goals is that some objectives are stated in the form of a normative dictum. For example, we must have 46 per cent of the market, or we must expend 6 per cent of our gross revenue in advertising.

Third, some objectives are stated in a nonoperational form. In other words, they are not necessarily in a form to have any effect on decisions. Thus, a nonoperational goal may be that the firm desires to be a leading innovator in the industry.

In sum, in the behavioral approach to the theory of the firm, we have to acquire a complete understanding of the business firm as an institution and it is necessary to analyze the effects of such variables as goal setting, goal adjustment, information flow , search patterns, and other internal organizational characteristics on the firm's decisions In general , the behavioral approach assumes that the firm is essentially an adaptively rational system. With this general assumption, the behavioral approach then investigates effects of variables internal to the firm on price and output decisions. This approach is in sharp contrast to the traditional theory which ignores the internal structure of the firm. The traditional theory assumes that market considerations dominate the internal structure of the firm in price and output decisions. This seems to a valid proposition in the case of perfect competition. In the case of oligopoly, however, the firm can affect market by its behavior and it is not completely dominated by market considerations. Thus a study of the internal factors affecting business decisions can increase our understanding of firm behavior. Especially in imperfect markets.

3.8 Conclusion

In what follows is that Baumol's Sales Maximization model assumes that a firm attempts to maximize its total revenue subject to a minimum profit constraint. The implications of this model differ in many respects from the implications of both traditional profit-maximizing model and Williamson utility-maximizing model. A revenue maximizing firm produces at an output where its MR is less than MC; both a utility-maximizing and a profit-maximizing firm equates MR with MC.

The growth maximization model, also a model of managerial enterprise developed by Marris is essentially concerned with the time path of expansion of the firm.Many firms under imperfect market structure follow a *mark-up or full-cost or cost-plus pricing strategy;* under this method they estimate AC and then add some fixed percentage to that cost to reach the price they charge. Behavioural theory of the firm on the other hand focuses on a different set of questions, questions concerning the internal decision-making structure of the firm.

3.9 Summary

Sales maximization: The sales maximization model suggests that firms maximize sales revenue to a minimum profit constraint. This model describes typical oligopoly behaviour.Here sales maximization is a proxy for the objective of the managers whose salary, prestige and promotion prospects are positively related to the sales reenue of the corporator. Furthermore, the managers are viewed as the effective decision-makers (especially over issues such as pricing and investment) and make decisions that enhance their objectives.

Comparison with profit maximization hypothesis : If this constrained maximization problem is solved by the choice of output level, output is higher under the sales-maximization model than under the profit-maximization model.

Marris model: Marris (1964) can be viewed as a developer of the work of Baumol, in which the objective of managers, and thereby, the corporation, is growth of sales maximization subject to a takeover constraint. Growth of corporate size is the aim of managers of corporations. Various developments in industrial economics brought forth a view that managerial behavior is to be growth –oriented, that is operating in monopolistic competitive environment. Managers can choose profits and growth combination subject to the constraint of the external sources of finance.

Williamson's model of firm behavior: O.E .Williamson's model of firm behavior (1963) focuses on the self-interest seeking behavior of corporate managers. The theory basically ignores the owner's interest whenever there is a dichotomy between owners and management. To this extent it goes beyond Baumol's hypothesis of sales maximization, where managers at least ensure some minimum profit for the owners].

Full-cost pricing rule: The pricing method used by many firms in situation of imperfect competition; under this method they estimate AC and then add some fixed percentage to that cost to reach the price they charge.

Behavioural theory of the firm : It focuses on a different set of questions, questions concerning the internal decision-making structure of the firm.

3.10 Exercises

A. Short –answer Type Questions

1. Why do firms pursue sales maximization objective even by sacrificing current profits?
2. What is the difference between profit maximization and sales maximization ?
3. What is full-cost pricing rule?
4. What is the focus of Williamson's model of firm behavior?
5. In order to formalize his model, Williamson introduces four different types of profits. What are they?
6. Williamson's managerial discretion model assumes that the managers of the firm are trying to maximize a utility function which depends upon three components. What are they?

7. What is behavioral theory of the firm? Where is it most applicable?

B. Medium-answer Type Questions

1. What would be effects on the output of a firm if the objective is maximization of sales subject to a profit constraint?
2. Do you agree that Marris Model of Managerial Enterprise is essentially a growth Maximization model ? give reasons for your answer.
3. Show that he optimal mark-up (m) is m $[e_p/(e_p+1)]-1$, where e_p is the price elasticity of demand for the product which usually carries a negative sign.
4. Describe the motivation behind the behavioral theory of firm.
5. The Behavioural Theory of the Firm are interested in answering some key questions. What are they?
6. How does the behavioral approach to the theory of the firm differ sharply from the traditional theory of the firm?

C . Long Answer Type Questions (At least Four Questions)

1. What are the major objectives to the profit maximizing hypothesis? What would be effects on the price and output of a firm if the objective of profit maximization is replaced by any of the alternative hypotheses?
2. The implications of Baumol's model differ in many respects from the implications of both Williamson's model and the traditional profit-maximization model. Discuss
3. Why the behavioural theory of the firm has been so important in today's oligopoly market?

3.11 References

Baumol, W.J.(1977). *Economic Theory and Operations Analysis,* PHI.

Baumol, W.J. (1967) *Business Behaviour, Value and Growth*. New York: Harcourt

Cohen K.L and Cyert, R.H. (1975). Theory of the Firm: Resource allocation in a Market Economy., Printice Hall.U.S.A. Read Ch 17 and 18

Gupta, 3B.S. (1990) *Managerial Economics,* Tata McGrawHill Publishing Company Ltd.

Marris, R. (1964). *The Economic Theory of Managerial Capitalism*. London: Macmillan

Williamson, O. E. (1964). *The economics of discretionary behavior: Managerial objectives in a theory of the firm*. Chapters 6-7 Prentice-Hall.

Unit 4 ❑ Game Theory

Structure

4.1 Objectives

After going through this unit, you will be able to :

- Define game theory and explain how it helps to better understand mutually interdependent decisions;
- Explain the difference between cooperative and non-cooperative games;
- Explain the concepts of strategies, different solution concepts of Nash Equilibrium under Complete Information in both the cases of static and dynamic games; and
- Have real life applications.

4.2 Introduction

The subject matter of game theory is exactly those interactions within a group of individuals (or governments, firms, etc.) where the actions of each individual have an effect on the outcome that is of interest to all. Yet, this is not enough for a situation to be a proper subject of game theory — the Game theory studies strategic interactions way that individuals act has to be strategic, i.e., they should be aware of the fact that their actions affect others. The fact that my actions have an effect on the outcome does not necessitate strategic behavior, if I am not aware of that fact. Therefore, we say that game theory studies strategic interaction within a group of individuals. By strategic interaction we mean that individuals know that their actions will have an effect on the outcome and act accordingly. Therefore, to bring some structure into the game, one has to introduce certain rules and assumptions in the game framework. The most important, assumption of game theory is that individuals are rational and have common knowledge of an outcome.

Definition of Rationality: An individual is rational if she has well-defined and consistent objectives (or preferences) over the set of possible outcomes and she implements the best available strategy to pursue them and that fact is known to her. A fact is, infact, common knowledge if everybody knows it, if everybody knows that everybody knows it, if everybody knows that everybody knows that everybody knows it, an so on.

The following are the building blocks of Game Theory :

- A **game** involves players making strategic decisions;
- **Players** are the decision-making units;
- An **action** is an option available to a player and a collection of actions make up a **strategy** set from where the player can choose;
- The **payoffs** that can accrue to each player is resulting from all possible plays of the game;

The *information* each player possesses, first about the game itself, and second, about what has happened in the course of playing the game. In a game of *complete information*, every player knows everything there is to know about the game in which they are engaged, including the characteristics, such as cost functions, of the other players. The Prisoner’s Dilemma type as discussed in the following section, are examples of games of complete information. There is *perfect information* if, when a player has to choose an action, she knows what actions, if any, have already been chosen up to that point by the other players (she is assumed always to be able to recall her own previous actions). The Stackelberg leader-follower game is a game of perfect information since firm *B*, being a follower, knows the output of *A* when it is its turn to choose an output level. The Prisoners' Dilemma game has *imperfect information* since neither player knows the output of the other when taking its decision. It is usual to make the *common knowledge assumption* : every player knows that the other players know the characteristics of the game, knows that they know she knows, and so on *ad infinitum*. A key difference between static and dynamic games of complete information is that in a static game no new information is revealed to any of the players during the game before they make their play contrary to dynamic games. The theory is presented in the following sections, corresponding to whether the game in question is static or dynamic and to whether it has its basis on complete or incomplete information. Under these classifications, there are two main ways of representing games. The *extensive form* uses a graphical device known as a game tree to illustrate the actions available to the players and the sequence in which they have to be chosen. The *strategic form*, also often called the *normal form*, collapses the extensive form of the game into a matrix, which shows the payoffs corresponding to the possible strategy choices of the two players.

4.3 Static Games of Complete Information

Complete information means that there is no private information : the timing, feasible moves and payoffs of the game are all common knowledge. Moving on, first, we have a **static two player, simultaneous-move game with complete information:**

1) Player 1 chooses an action a_1 from a set of feasible actions A_1. Simultaneously, player 2 chooses an action a_2 from a set of feasible actions A_2.

2) After the players choose their actions, they receive payoffs: $u_1(a_1, a_2)$ to player 1 and $u_2(a_1, a_2)$ to player 2.

In a game of complete information, the players' payoff functions are common knowledge. There are different ways in which one should play the game keeping the rationality concept in mind.

Definition : Let Action a_1' is **dominated** by action a_1'' for player 1 if and only if for whatever action player 2 chooses, player 1 gets a higher payoff from playing a_1''

than from playing $a_1{'}$. That is, $u_1(a_1', a_2) < u_1(a_1'', a_2)$ for each given action a_2 in 2's action set, A_2. Note that a rational player will not play a dominated action. In Table 1, an example is presented.

Table 1 : Iterated Elimination of Dominated Strategies using a Normal Form representation

		Player 2		
		Left	Middle	Right
Player 1	Up	1, 0	1, 2	0, 1
	Down	0, 3	0, 1	2, 0

Source : Gibbons (1997)

Consider the game in Figure 1. Player 1 has two actions, {Up, Down]; player 2 has three, {Left, Middle, Right]. For player 2, playing Right is dominated by playing Middle: if player 1 chooses Up, then Right yields 1 for player 2, whereas Middle yields 2; if 1 chooses Down, then Right yields 0 for 2, whereas Middle yields 1. Thus, a rational player 2 will not play Right.' Now take the argument a step further. If player 1 knows that player 2 is rational, then player 1 can eliminate Right from player 2's action space. That is, if player 1 knows that player 2 is rational, then player 1can play the game as if player 2's only moves were Left and Middle. But in this case, Down is dominated by Up for player 1: if 2 plays Left, then Up is better for 1, and likewise if 2 plays Middle. Thus, if player 1 is rational (and player 1 knows that player 2 is rational, so that player 2's only moves are Left and Middle), then player 1 will not play Down. Finally, take the argument one last step. If player 2 knows that player 1 is rational, and player 2 knows that player 1 knows that player 2 is rational, then player 2 can eliminate Down from player 1's action space, leaving Up as player 1's only move. But in this case, Left is dominated by Middle for player 2, leaving (Up, Middle) as the solution to the game. This argument shows that some games can be solved by (repeatedly) asking how one should not play the game.

4.4 Solution Concepts— Pure and mixed strategy, Nash applications

We have just seen that asking how one should not play a given game can shed some light on how one should play. To introduce Nash equilibrium, we take a similarly indirect approach : instead of asking what the solution of a given game is (that is, what all the players should do), we ask what outcomes cannot be the solution. After eliminating some outcomes, we are left with one or more possible solutions.

Suppose game theory offers a unique prediction about the play of a particular game. For this predicted solution to be correct, it is necessary that each player be willing to choose the strategy that the theory predicts that individual will play. Thus, each player's predicted strategy must be that player's best response to the predicted strategies of the other players. Such a collection of predicted strategies could be called "strategically stable" or "self-enforcing," because no single player wants to deviate from his or her predicted strategy. We will call such a collection of strategies a '**Nash equilibrium**'.

To relate this definition to the motivation above, suppose game theory offers the actions (a_1^* , a_2^*) as a solution. Saying that (a_1^* , a_2^*) is not a Nash equilibrium is equivalent to saying that either a_1^* is not a best response for player 1 to a_2^*, or a_2^* is not a best response for player 2 to a_1^*, or both. Thus, if the theory offers the strategies (a_1^* , a_2^*) as the solution, but these strategies are not a Nash equilibrium, then at least one player will have an incentive to deviate from the theory's prediction, so the prediction is not true. Moving on from the technical stuff, we analyze a few games which can have either a unique Nash or multiple Nash Equilibria. In Table 2, there are multiple Nash equilibria while the game in Table 3 has a unique Nash.

Table 2 : The Battle of the Sexes Game

	Opera	Football
Opera	3.2	1.1
Football	0.0	2.3

In game theory, **battle of the sexes** (**BoS**) is a two-player coordination game. Imagine a couple that agreed to meet this evening, but cannot decide if they will be attending the opera or a football game. The husband would prefer to go to the football game. The wife would rather go to the opera. Both would prefer to go to the same place rather than different ones. If they cannot communicate, where should they go? If Husband decides to go to Opera, it is in the best interest of the Wife to go to Opera as well. Similarly, if the Husband decides to go to Football, the Wife also decides to go to Football. Similarly, if Wife decides to go to Opera, the Husband decides to go to Opera as well. Same for the case where the Wife decides to go to Football and

Husband does the same. So the two Nash Equilibria are (Opera, Opera) and (Football, Football) but no obvious way to decide between these equilibria.

Table 3 : The Prisoners' Dilemma

	Player B	
Player A	Confess	Deny
Confess	-3, -3	0, -6
Deny	-6, 0	-1, -1

The pay-offs in the **Prisoners' Dilemma** game are the term of jail sentences each player has been convicted with. To solve the game, let us say, if Player B decides to deny committing the crime, then Player A will be better off by confessing, since she will get off free.

Similarly, if Player B confesses, then, confessing is a best option as a sentence of 3 months is better than a sentence of 6 months. Thus, whatever Player B does, Player A is better off confessing. The exactly same thing goes for Player B – she is better off confessing as well. Therefore, the unique Nash equilibrium for this game is for both players to confess. The paradox in such a game lies in the fact that if the prisoners were allowed to communicate they could easily chosen not to confess leading to minimum years of punishment. This actually sums up that with asymmetry of information, players end up choosing a sub-optimal Nash like (Confess, Confess). However, Prisoners' Dilemma in a real life situation might have more than two players as popularised by Henry J. Hamburger and Thomas C. Schelling. It is an interactive decision involving three or more players who each face a choice between a cooperative strategy labelled *C* and a non-cooperative or defecting strategy labeled *D*. The payoff structure is such that *D* is a dominant strategy for each player in the sense that each player obtains a better payoff by choosing *D* than *C* no matter how many of the other players choose *C*.

Let us now apply the concept of Nash equilibrium to a **Bertrand game of price competition**. Notice first that the strategy high (price) is dominated by the strategy medium (and indeed this is true for both the firms). Hence, we can eliminate high as an irrational strategy for both firms (it either leads to no sales or a 50% share of a small market).

Having eliminated high we are left with the following payoff matrix.

Firm 1 \ Firm 2	high	medium	low
high	6, 6	0, 10	0, 8
medium	10, 0	5, 5	0, 8
low	8, 0	8, 0	4, 4

Firm 1 \ Firm 2	medium	low
medium	5, 5	0, 8
low	8, 0	4, 4

We can now see that low dominates the medium price. Hence, the outcome to Iterated Elimination of Dominated Strategies is (low, low). Notice that medium is a useful strategy only if you believe that your opponent is going to price high; hence, once you are convinced that he will never do so, you have no reason to price medium either. In a similar context, **Cournot Competition** describes an industry structure (i.e. an oligopoly) in which competing companies simultaneously (and independently) choose a quantity to produce, i.e. **quantity competition**. Each company has to consider the expected quantity supplied by its competitors to maximize their own profits. Neither model is necessarily "better" than the other. The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation. If capacity and output can be very easily changed, Bertrand is generally a better model of duopoly competition. If output and capacity are difficult to adjust in relation to prices, then Cournot is generally a better model.

Definition : In the two-player, simultaneous-move game, the action $\left(a_1*, a_2*\right)$ are a **Nash equilibrium** if a_1* is best response for player 1 to a_2*, and a_2* is a best response for player 2 to a_1*. That is, must satisfy for every in $A_{1,}$ and must satisfy for every in $A_{2.}$

In any game in which each player would like to outguess the other, there is no pair of strategies satisfying the definition of Nash equilibrium given above. Instead, the solution to such a game necessarily involves uncertainty about what the players will do.

To model this uncertainty, we will refer to the actions in a player's action space (A) as pure strategzes, and we will define a mixed strategy to be a probability distribution over some or all of the player's pure strategies.

Definition : A **mixed strategy** is a probability distribution over some or all of the player's pure strategies.

Result (Nash, 1950) : *Any game with a finite number of players, each of whom has a finite number of pure strategies, has a Nash equilibrium (possibly involving mixed strategies).*

Table 4 : The Matching Pennies Game

		Player 2	
		Heads	Tails
Player 1	Heads	–1, 1	1, –1
	Tails	1, –1	–1, 1

This game has no pure strategy Nash equilibrium since there is no pure strategy (heads or tails) that is a best response to a best response. In other words, there is no pair of pure strategies such that neither player would want to switch if told what the other would do. Instead, the unique Nash equilibrium of this game is in mixed strategies : each player chooses heads or tails with equal probability. That is, to calculate the equilibrium point in this game, note that a player playing a mixed strategy must be indifferent between his two actions (otherwise he would switch to a pure strategy). This gives us two equations :

For Player 1, the expected payoff when playing Heads is -1.x + 1.(1 - x) and for playing tails is 1.x – 1.(1 – x), i.e. x = 0.5.

For Player 2, the expected payoff when playing Heads is -1.y + 1.(1 - y) and for playing tails is 1.y – 1.(1 – y), i.e. y = 0.5.

In this way, each player makes the other indifferent between choosing heads or tails, so neither player has an incentive to try another strategy.

4.5 Dynamic Games of Complete Information and Solution Concept

Definition : Dynamic games with complete information:

1) Player 1 chooses an action a_1 from a set of feasible actions A_1.
2) Player 2 observes 1's choice and then chooses an action a_2 from a set of feasible actions A_2.
3) After the players choose their actions, they receive payoffs: $u_1(a_1, a_2)$ to player 1 and $u_2(a_1, a_2)$ to player 2.

A classic example of a dynamic game with complete information is Stackelberg's (1934) sequential-move version of Cournot duopoly. The new solution concept in this section is backward induction. We will see that in many of these dynamic games there are many Nash equilibria, some of which depend on non-credible threats — defined as threats that the threatener would not want to carry out, but will not have to carry out if the threat is believed. Backward induction identifies a Nash equilibrium that does not rely on such threats.

Note : *The backward-induction solution to a game is always a Nash equilibrium that does not rely on non-credible threats or promises.*

Definition : A **subgame** is a piece of an original game that remains to be played beginning at any point at which the complete history of the play of the game, thus far, is common knowledge to the players.

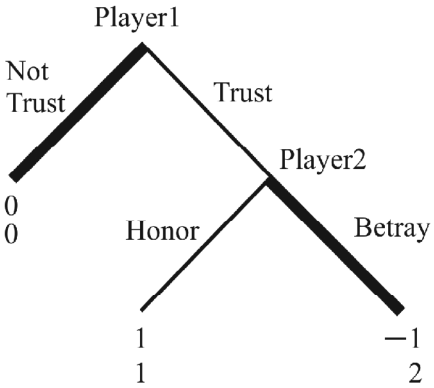
Definition : A Nash equilibrium (of the game on the whole) is a **subgame-perfect Nash equilibrium** if the players' strategies constitute a Nash equilibrium in every subgame.

Result: *Any finite game has a subgame-perfect Nash equilibrium, possibly involving mixed strategies, because each subgame is itself a finite game and hence has a Nash equilibrium.*

4.6 Backward Induction

Given the concept of backward induction, we move to solving a few examples in this regard.

Table 5 : The Trust Game

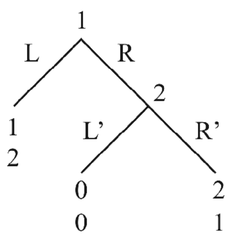


		Player2	
		Honor	Betray
Player1	Trust	1,1	−1,2
	Not Trust	0,0	0,0

Consider the Trust Game in table 5, in which player 1 first chooses either to Trust or Not Trust player 2. For simplicity, suppose that if player 1 chooses Not Trust then the game ends Player 1 terminates the relationship. If player 1 chooses to Trust 2, however, then the game continues, and 2 chooses either to Honor or to Betray 1's trust. If player 1 chooses to end the relationship, then both players' payoffs are 0. If 1 chooses to Trust 2, then both players' payoffs are 1 if 2 Honors 1's trust, but player 1 receives -1 and player 2 receives 2 if player 2 Betrays 1's trust. All of this is captured by the game tree on the left-hand side of Table 5. The game begins with a decision node for player 1 and reaches a decision node for player 2 if 1 chooses Trust. At the end of each branch of the tree, player 1's payoff appears above player 2's.

We solve the Trust Game by working backward through the game tree. If player 2 gets to move (that is, if player 1 chooses Trust) then 2 can receive a payoff of 1 by choosing to Honor 1's trust or a payoff of 2 by choosing to Betray 1's trust. Since 2 exceeds 1, player 2 will Betray 1's trust. Knowing this, player 1's initial choice amounts to ending the relationship (and so receiving a payoff of 0) or Trusting player 2 (and so receiving a payoff of -1, after player 2 Betrays 1's trust). Since 0 exceeds -1, player 1 should Not Trust. These arguments are summarized by the bold lines in the game tree.

Table 6 : Ruling out Non-Credible Threats



	L'	R'
L	1,2	1,2
R	0,0	2,1

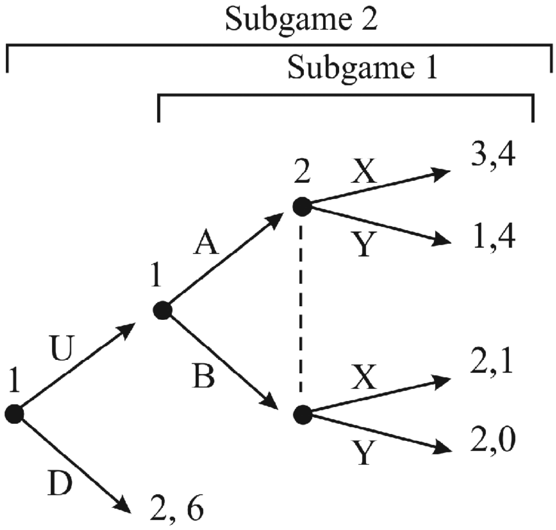
Working backward through this game tree shows that the backward-induction solution is for player 2 to play R' if given the move and for player 1 to play R. But the normal form reveals that there are two Nash equilibria: (R, R´) and (L, L´). The second Nash equilibrium exists because player 1's best response to L' by 2 is to end the game by choosing L. But (L, L´) relies on the noncredible threat by player 2 to play L' rather than R' if given the move. If player 1 believes 2's threat, then 2 is off the hook because 1 will play L, but 2 would never want to carry out this threat if given the opportunity. So, herein, comes the refinement and the action combination of (L, L´) can be easily ruled out.

Backward induction can be applied in any finite-horizon game of complete information in which the players move one at a time and all previous moves are common knowledge before the next move is chosen. The method is simple: go to the end of the game and work backward, one move at a time.

4.7 Sub-game Perfect Nash

Sub-game-perfect Nash equilibrium is a refinement of Nash equilibrium; that is, to be subgame-perfect, the players' strategies must first be a Nash equilibrium and must then fulfill an additional requirement. The point of this additional requirement is, as with the case of backward induction, to rule out Nash equilibria that rely on non-credible threats.

Table 7a : Sub-game Perfect Nash



1 \ 2	X	Y
UA	3,4	1,4
UB	2,1	2,0
DA	2,6	2,6
DB	2,6	2,6

To provide an informal definition of subgame-perfect Nash equilibrium, we return to the motivation for Nash equilibrium-namely, that a unique solution to a game-theoretic problem must satisfy Nash's mutual-best-response requirement. In many dynamic games, the same argument can also be applied to certain pieces of the game, called subgames. A subgame is the piece of an original game that remains to be played beginning at any point at which the complete history of the play of the game thus far is common knowledge.

To solve this game, first find the Nash Equilibria by mutual best response of Subgame 1. The first normal-form game is the normal form representation of the whole extensive-form game. Based on the provided information, (UA, X), (DA, Y), and (DB, Y) are all Nash equilibria for the entire game. The second normal-form game is the normal form representation of the subgame in Table 7b starting from Player 1's second node with actions A and B.

For the second normal-form game, the Nash equilibrium of the subgame is (A, X). For the entire game Nash equilibria (DA, Y) and (DB, Y) are not subgame perfect equlibria because the move of Player 2 does not constitute a Nash Equilibrium because it is not going to be played by any rational individual. The Nash equilibrium (UA, X) is subgame perfect because it incorporates the subgame Nash equilibrium (A, X) as part of its strategy.

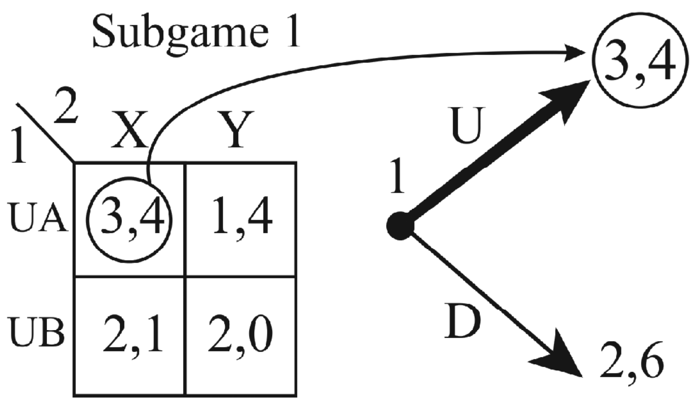


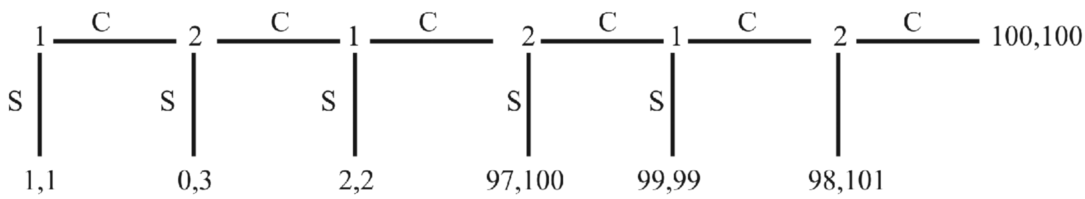
Table 7b : Solving the Sub-game of an Entire Game

Then use backwards induction and plug in (A, X) $\rightarrow$ (3, 4) so that (3, 4) become the payoffs for Subgame 2 i.e. for the entire game. The dashed line indicates that player 2 does not know whether player 1 will play A or B in a simultaneous game. In Subgame 1, we have the Nash Equilibrium as (3, 4). Player 1 chooses U rather than D because

3 > 2 for Player 1's payoff so (DA) and (DB) goes out the picture. The resulting equilibrium is (A, X) $\rightarrow$ (3, 4).

4.8 Applications

1. Centipede Game: Two players, 1 and 2, take turns choosing one of two actions each time, continue or stop. They both start with Rs. 1 in their respective piles, and each time 1 says continue, Rs. 1 is taken away from his pile, and Rs. 2 are added to the other player's pile. The game automatically stops when both players have Rs. 100 in their respective piles.



Solution : Backward induction implies that a player should say stop whenever it is his turn to move. In particular, Player 1 should say stop at the very first node, and both players leave with just the Rs. 1 they start out with. If the player who moves at the end, i.e. Player 2 acts rationally , will never come to the last node as she will stop the game in the second last node where Player 2 earns a pay-off of 101 which is > 100. Now Player 1 has the move, she will not allow the second last node to occur as she herself can earn more by stopping the game at the third last node, i.e. 99 > 98. In this way, assigning the payoff vector associated with this move to the node at hand, we delete all the moves stemming from this node so that we have a shorter game, where our node is a terminal node. Repeat this procedure until we reach the origin i.e. both players get Rs. 1.

2. In a business firm :

	Firm B	
Firm A	Advertise	Don't Advertise
Advertise	10, 5	15, 0
Don't Advertise	6, 8	10, 2

What strategy should each firm choose? First consider Firm A. It should clearly advertise because no matter what firm B does, Firm A does best by advertising. If Firm B advertises, A earns a profit of 10 if it advertises but only 6 if it doesn't. If B does not advertise, A earns 15 if it advertises but only 10 if it doesn't. Thus advertising is a dominant strategy for Firm A. The same is true for Firm B : No matter what firm A does, Firm B does best by advertising. Therefore, assuming that both firms are rational, we know that the outcome for this game is that both firms will advertise.

3. Tit – for – Tat Strategy : Firm 1 says, I start out with a high price, which I maintain so long as you continue to "cooperate" and also charge a high price. As soon as you lower your price, however, I follow suit and lower mine. If you later decide to cooperate and raise your price again, I'll immediately raise my price as well.

		Firm 2	
		Low price	High price
Firm 1	Low price	10, 10	100, – 50
	High price	–50, 100	50, 50

Case Study for Prisoners' Dilemma in the Climate Change Scenario

The Prisoners' dilemma is a game in the game theory that shows why two rational individuals might not cooperate even if it is in their best interests to do so. The rules of the game are as follows: two persons are in two separate cells, unable to talk to each other. They each know that if they both keep silent, they will both be free. But the guards want each prisoner to 'spill the beans' on his accomplice. The guards keeping each prisoner that the other prisoner is about to talk, how terrible the punishment will be if he does not speak up first. If prisoner A talks and prisoner B is silent, the prisoner A gets off and gets parks, while prisoner B goes to jail for five years, and vice versa. However, if both talk, they both go to jail for two years. In the game and in real life, most of the time, both prisoners talk, even though know very well if they had remained quiet, they would both have gotten off scot –free.

Climate change is a global prisoners' dilemma game with horns and on steroids. If we, the world, cooperate or arrest climate change, we get a wonderful place to live in. If we do not, for some of us, our way of life ends. But wait, there is more. If we don't cooperate, we don't all go to the same kind of prison—some of us stay free, some go to a low-security prison, and some go to hell. Important point is that climate change impacts the different players –i.e., countries—very differently. Understanding the differential "impacts" of climate change will help us understand why so little has been done so far, where " impact" is defined by the IPCC as the intersection of risk, vulnerability and adaptation. An example may help us to understand what "impact" means. A temperature of 38^0C is manageable if one can stay indoors in AC. However, if one performs a outdoor manual job, such days could prove fatal. The former is low-impact, while the latter is high impact. Wealthy countries can afford to adapt or mange the ravages of harsh climate. For them, the impact is lower than for, say, the Indian states of Telengana or Odisha.

Sums on Game Theory:

1. Solve the Nash Equilibrium of the following game :

		Player 2	
		L_2	R_2
Player 1	L_1	1, 1	5, 0
	R_2	0, 5	4, 4

If Player 1 plays R, it is in the best interest of Player 2 to play R a payoff of 5 is greater than a payoff of 4. On a similar note, if Player 2 plays L, Player 1 plays L and if Player 2 plays R, Player 1 plays R. So find out the Nash?

[**Ans.** Hints : If Player 1 plays L, it is in the best interest of Player 2 to play L as well given a payoff of 1 is greater than a payoff of 0.]

2. Solve for the pure strategy Nash Equilibrium using the Peace-War payoff matrix

	Country B	
Country A	Peace	War
Peace	3, 3	0, 4
War	4, 0	1, 1

[**Hints :** Similar to Question 1]

3. Using the payoff matrix in Q2, what are the pure strategy Nash equilibria (NE) of this game? Justify your answer**.**

[Answers:

The two pure NE are:

- (B, C): If Shelia plays B, then Thomas' best response is C with a payoff of 20 rather than 12 if Thomas played D; if Thomas plays C, then Shelias's best response is B with a payoff of 15 rather than 10 if she plays A.
- (A, D): Similar reasoning shows that (A, D) is a pure NE.

The worst NE is (B, C) with SW of 35 while the optimum outcome is (A, D) with SW of 38.]

4. Using the payoff matrix in Q2, this game has a fully mixed strategy Nash equilibrium in which both Shelia and Thomas play each of their actions with positive probability. What are the mixed strategies for each player in this equilibrium? Show how you would compute such a mixed equilibrium and verify that your mixed strategies are indeed in equilibrium.

[Answer:

Let Thomas play C with probability p and hence play D with probability (1 – p). By the principle of indifference, it must be that the expected payoff for Shelia is the same whether she plays A or B. Thus:

$$10p + 15(1 - p) = 14p + 6(1 - p)$$

so that $p = \frac{8}{13}$

Let Shelia play *A* with probability *q* and hence plays *B* with probability (1 – *q*). In order to insure that Thomas' payoff's are the same for strategies *C* and *D*, we have that :

$$q(16) + (1 - q)(20) = q(24) + (1 - q)(12)$$

so that $q = \frac{1}{2}$.

To verify that $p = \frac{8}{13}, q = \frac{1}{2}$ is a mixed NE, we calculate

- Shelia's payoff for A is $\frac{8}{13}(10) + \frac{5}{13}(14) = \frac{150}{13}$
- Shelia's payoff for B is $\frac{8}{13}(15) + \frac{5}{13}(6) = \frac{150}{13}$
- Thomas's payoff for C is $\frac{1}{2}(16) + \frac{1}{2}(20) = 18$
- Thomas's payoff for D is $\frac{1}{2}(24) + \frac{1}{2}(12) = 18$

5. Explain the solution for the game given below (using backward induction methodology)

Two players, 1 and 2, take turns choosing one of two actions each time, continue or stop. They both start with Rs. 1 in their respective piles, and each time 1 says continue, Rs. 1 is taken away from his pile, and Rs. 2 are added to the other player's pile. The game automatically stops when both players have Rs. 100 in their respective piles.

6. Consider the following game in matrix form with two players. Payoffs for the row player Shelia are indicated first in each cell, and payoffs for the column player Thomas are second.

	C	D
A	10, 16	14, 24
B	15, 20	6, 12

(a) Does either player have a dominant strategy? Explain your answer.

[**Answer:** Neither player has a dominant strategy. For example, if Shelia plays A and Thomas plays D then Shelia's payoff is 14. But if Shelia plays B and Thomas plays C, then Sheilas's payoff is 15. A similar argument shows that Thomas also does not have a dominant strategy.]

7. Suppose that Christie's and Sotheby's are considering offering guarantees that the paintings they sell are not forgeries. The payoff matrix below shows the profits for each firm under the four possible profiles.

		Christie's: Guarantee	Christie's: No guarantee
Sotheby's	Guarantee	1 (Christie's), 1 (Sotheby's)	0 (Christie's), 3 (Sotheby's)
Sotheby's	No guarantee	3 (Christie's), 0 (Sotheby's)	2 (Christie's), 2 (Sotheby's)

(a) Do the auction houses have dominant strategies?

(b) Is there a Nash equilibrium?

(c) Is this game a prisoners' dilemma?

[Answer:

(a) Yes, providing a guarantee is a dominant strategy for both firms.

(b) Yes, the upper left cell, where both firms provide guarantees is a Nash equilibrium.

(c) Yes, the game is a prisoners' dilemma because the payoff in equilibrium is less than in the lower right cell for both firms.]

8. Consider a game with the following payoff matrix. Find a Nash equilibrium in mixed strategies.

		Player: A	Player: B
Player 2	A´	0 (upper), 1 (lower)	2 (upper), 0 (lower)
Player 2	B´	1 (upper), 0 (lower)	0 (upper), 2 (lower)

[Answer:

If player 1 chooses A with probability *p* and *B* with probability 1 — *p*, the expected payoff for player 2 is *p* if he chooses *A'* and 2(1 — *p)* if he chooses *B'*. Player 2 is indifferent among *A'*, *B'*, and mixed strategies when *p* = 2(1 — p), that is, when *p* = 2/3. Similarly, if player 2 chooses *A'* with probability *q* and *B'* with probability (1 — q), the expected payoff for player 1 is 1 — q if she chooses *A* and *2q* if she chooses *B*. She is indifferent among *A*, *B*, and mixed strategies when 1 — *q* = *2q*, that is, when *q* = 1/3. Thus the Nash equilibrium is a pair of mixed strategies with *p* = 2/3 and *q* = 1/3.]

4.9 Conclusion

Game theory is the study of the choice of strategies by interacting rational agents to discover which strategy is a person's best response to the strategies chosen by the others. Here, we have basically looked at non-cooperative game theory where two players or competitors with two or more strategic options lock horns. So the strategy of a particular company or industry shows their response through strategic moves, the dominant strategy can be adopted easily if you are aware about the response of your competitors. Game theory is based on a scientific metaphor, the idea not only considers a game but also considers economic competition, war and elections, etc. and can be treated and analyzed thereof.

4.10 Summary

- **Game theory**: originally developed by von Neumann and Morgenstern with the publications of their magisterial *Theory of games and Economic Behaviour* in 1944 focuses on the strategic interaction of rational individuals who know each other to be rational. Rationality is interpreted as maximizing expected utility. Game theorists have interpreted a variety of strategic interactions, including cooperative and non-cooperative games, simultaneous and sequential choice, single and repeated games, and pure and mixed strategies. Many of their concepts have been applied in modern theories of oligopoly. Game theory attempts to study decision making in situations in which there are mixtures of conflict and cooperation .Game theory sheds insights about decision making in markets that are oligopolies. A game's outcome depends on the strategies used by each player. These possible outcomes are known as payoffs.

- A **payoff matrix** is a table that describes the outcome for each player and for each set of strategic choices.
- A **dominant strategy (DS)** is a strategy that produces the optimal outcome regardless of what the other players do. In other words , a dominant strategy for a player is a strategy that produces the best results for that player regardless of the strategy chosen by his opponent.
- A **dominant strategy equilibrium (DSE)** occurs if each player in a game chooses its dominant strategy.
- A **Nash equilibrium** occurs if every player's strategy is optimal given its competitors' strategies ,or a Nash equilibrium is a strategy combination for which no player has an incentive to be the only player to switch to another strategy.
- A **cooperative game** is a game in which the players can negotiate explicit binding contracts.
- **A non-cooperative game** is a game in which formal negotiation and entering into a legally binding contract is not possible.
- A **zero sum game** is a game where the sum of payouts is constant (i.e. zero).
- A **mixed strategy** is a probability distribution over some or all of the player's pure strategies. A lottery over pure strategies.
- The **backward-induction** solution to a game is always a Nash equilibrium that does not rely on non-credible threats or promises.
- A **subgame** is a piece of an original game that remains to be played beginning at any point at which the complete history of the play of the game, thus far, is common knowledge to the players. A Nash equilibrium (of the game on the whole) is a **subgame-perfect Nash equilibrium** if the players' strategies constitute a Nash equilibrium in every subgame.

The refinement of Nash's equilibrium concept that did gain wide acceptance among game theorists is called subgame perfect equilibrium. Due to ground –

breaking efforts of Reinhard Selten, subgame perfection is now the accepted measure of rational play in dynamic (extensive form) game of complete information.

- **Prisoner's Dilemma**: a game with two strategies (cooperate or defect) for each of the two players, each player having incentives to defect but both being better off if both cooperate. The crux of the dilemma is that each individual, faced with the uncertainty of how others will behave, may be led to adopt a course of action that proves to be detrimental for all those individuals making the same decision. A strong coalition might have led to a solution preferred by everyone in the group.

  Prisoner's dilemma is a situation which depicts the movements of the participants for a particular motive. Basically there are two or more players who try to move to different segments to achieve profit than their compotators or other participants. This situation would be more clear with the assistance of 2 by 2 matrix games. They have the minimal configuration necessary to be a game, which are two player with each two decision possibilities. These games can easily be examined and displayed in a matrix. The most famous 2-by-2 matrix example in this context is the Prisoner's dilemma. It is one of the most interesting games which game theory has to offer and at the same time deals with one of the most interesting thing of social science like the interaction of individual and society. Two criminals commit a bank robbery and are arrested by police. The police do not have sufficient evidence to prove their crime. In this situation they use game theory to prove it. Basically they separated them in two cells and visit each of them and offer the same deal. If you confess and your accomplice remains silent, he gets the full 5 year punishment and you go to free as principal witness. If he confesses and you remain silent, you get the full 5 year punishment and he goes free. If you both stay silent, all we can do is give you both 1 year for a minor charge, which is illicit possession of firearms. If you both confess, you each get 4 years.

  The prisoner's dilemma is the prototype of many economic problems in which individual's rationality in an uncertain situation can lead to an outcome that is collectively irrational.

4.11 Exercises

A. Short-answer Type Question

1. Explain the idea of Prisonner's Dilemma.
2. Explain the concept of Sub-game Perfect Nash Equilibrium.
3. What are the building blocks of Game Theory?
4. Discuss the concept of dominated strategy.
5. What do you mean by mixed strategy?
6. What is backward induction?
7. What is game theory? Is it used other fields other than economics itself? Give arguments.

B. Medium-answer type Questions

1. Solve the Nash Equilibrium of the following game :

Player 2

		L_2	R_2
Player I	L_1	1, 1	5, 0
	R_1	0, 5	4, 4

2. Solve for the pure strategy Nash Equilibrium using the Peace-War payoff matrix

	Country B	
Country A	Peace	War
Peace	3, 3	0, 4
War	4, 0	1, 1

3. Explain the solution for the game given below (using backward induction methodology)

Two players, 1 and 2, take turns choosing one of two actions each time, continue or stop. They both start with Rs. 1 in their respective piles, and each time 1 says continue, Rs. 1 is taken away from his pile, and Rs. 2 are added to the other player's pile. The game automatically stops when both players have Rs. 100 in their respective piles.

4. Consider the following game in matrix form with two players. Payoffs for the row player Shelia are indicated first in each cell, and payoffs for the column player Thomas are second.

	C	D
A	10, 16	14, 24
B	15, 20	6, 12

(a) Does either player have a dominant strategy? Explain your answer.

5. Using the payoff matrix in Q2, what are the pure strategy Nash equilibria (NE) of this game? Justify your answer.

6. Using the payoff matrix in Q2, this game has a fully mixed strategy Nash equilibrium in which both Shelia and Thomas play each of their actions with positive probability. What are the mixed strategies for each player in this equilibrium? Show how you would compute such a mixed equilibrium and verify that your mixed strategies are indeed in equilibrium.

7. Suppose that Christie's and Sotheby's are considering offering guarantees that the paintings they sell are not forgeries. The payoff matrix below shows the profits for each firm under the four possible profiles.

		Christie's	
		Guarantee	No guarantee
Sotheby's	Guarantee	1 1	0 3
	No guarantee	3 0	2 2

(a) Do the auction houses have dominant strategies?

(b) Is there a Nash equilibrium?

(c) Is this game a prisoners' dilemma?

8. Consider a game with the following payoff matrix. Find a Nash equilibrium in mixed strategies.

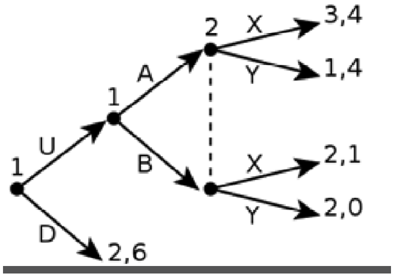
		Player A	Player B
Player 2	A´	0 1	2 0
	B´	1 0	0 2

A. Long-answer type Questions

1. Two breakfast cereal companies face a market in which two new variations of cereal can be successfully introduced. But, each firm has the resource to introduce only one product. From the product choice problem given below, find the Nash and discuss in details.

		Firm 2: Crispy	Firm 2: Sweet
Firm 1	Crispy	–5, –5	10, 10
	Sweet	10, 10	–5, –5

2. From the game tree given below, write down the normal form representation of the game and solve the Sub-game Perfect Nash Equilibrium.



(**Hints :** Normal Form :

1 \ 2	X	Y
UA	3,4	1,4
UB	2,1	2,0
DA	2,6	2,6
DB	2,6	2,6

3. Explain using backward induction a 2 player trust game. Highlight in the decision tree the optimal strategies the players choose.

4. From the case study on 'coordination' problem given below construct the pay-off matrix and solve for the pure/mixed strategy Nash Equilibrium.

   Consider two technology giants who are deciding between introducing a radical new technology in memory chips that could earn them hundreds of millions in profits (600 million), or a revised version of an older technology that would earn them much less (150 million). If only one company decides to go ahead with the new technology, rate of adoption by consumers would be much lower, and as a result, it would earn less (i.e. only 150 million) than if both companies decide on the same course of action (600 million each for new technology and 300 million each for old technology).

   [**Hint :** The payoff matrix is shown below (figures represent profit in millions of dollars).

	Company B	
Company A	New Technology	Old Technology
New Technology	600, 600	0, 150
Old Technology	150, 0	300, 300

4.12 References

Dutta, P. K. (1999). *Strategies and games: theory and practice.* Massachusetts : MIT press.

Hal R.Varian. *Intermediate Microeconomics : A Modern Approach*, W. W. Norton & Company, Inc, Eighth International Student Edition, 2014.

H. Gravelle and R. Rees. *Microeconomics*, Pearson Education, Third Edition, 2004.

F. Cowell. *Microeconomics*, Oxford University Press, 2006.

C. Snyder and W. Nicholson. *Fundamentals of Microeconomics*, Cengage Learning (India), 2010.

R. Gibbons. *Game Theory for Applied Economists*, Princeton University Press, 1992.

R. Gibbons. "An introduction to applicable game theory." *Journal of Economic Perspectives*, 11, no. 1 (1997) : 127-149.

Unit 5 ❑ General Equilibrium and Welfare

Structure

5.1 Objectives

After through this unit you will be able to :

- know choices available to a society when optimum welfare is not achievable.
- understand various criteria of compensation principle.
- apprehend equity efficiency trade-off as limits to redistribution.
- to provide a theoretical tool of understanding the economy entirely, the mechanics of its working, structure, the major forces making it work and the inter-dependence across sectors.

5.2 Introduction

Addressing the consumer and producer's objectives in the preceding analysis, we have considered only so-called partial equilibrium. Note that the attribute "partial' refers to looking at an equilibrium result in one market for a particular good only. In this method the analysis of equilibrium in the in the market for a particular commodity is carried out in isolation from other markets. It is just like a scenario you have worked out where the increased demand for agricultural, for example,products due to an increased income only and nothing has happened to other activities in the agricultural sector. That is to say, the impact of changed demand on markets such as inputs and employment has not come into effect. See that these other markets will also experience the change, which has been overlooked by us.

General equilibrium (GE) analysis seeks to determine equilibrium prices simultaneously in all markets.A model that includes the interdependencies of all the markets in the economy can account for the fact that if the equilibrium price in one market changes, the equilibrium prices and hence quantities in other markets are also affected. To understand such dimensions, we need a model that can accommodate the interactions of all the markets simultaneously and determine the properties of equilibria in all the markets. We have to develop a general equilibrium model, in contrast to the partial equilibrium models used thus far.

5.3 Concept of General Equilibrium

In economics, general equilibrium theory attempts to explain the behavior of supply, demand, and prices in a whole economy with several or many interacting markets, by seeking to prove that the interaction of demand and supply will result in an overall general equilibrium. General equilibrium theory contrasts to the theory of *partial* equilibrium, which only analyzes single markets.

General equilibrium theory both studies economies using the model of equilibrium pricing and seeks to determine in which circumstances the assumptions of general equilibrium will hold. The theory dates to the 1870s, particularly the work of French economist Léon Walras of the University of Lausanne in his pioneering 1874 work *Elements of Pure Economics*. This has been discussed in the following section.

General equilibrium analysis deals with the equilibrium of the whole organization in the economy where consumers, producers, resource-owners, firms and industries are involved. Not only should individual consumers and firms be in equilibrium in themselves but also in relation to each other.

5.4 Walras' Law

The term "Walras's law" was coined by Oskar Lange. It is named after the economist Léon Walras of the University of Lausanne who formulated the concept in his Elements of Pure Economics of 1874. Walras's law states that the sum of the values of excess demands across all markets must equal zero, whether or not the economy is in a general equilibrium. This implies that if positive excess demand exists in one market, negative excess demand must exist in some other market.

Or, in other words, it proves the existence of excess supply in one market must be matched by excess demand in another market so that it balances out. Mathematically,

$$\sum_{j=1}^{k} p_j \cdot D_j - \sum_{j=1}^{k} p_j \cdot S_j = 0$$

where p_j is the price of good j and D_j and S_j are the demand and supply respectively of good j.

This implies that if positive excess demand exists in one market, negative excess demand must exist in some other market. Thus, if all markets but one are in equilibrium, then that last market has to be in equilibrium.

This last implication is often applied in formal general equilibrium models. In particular, to characterize the general equilibrium position in a model where there are n markets and $n - 1$ of these are in equilibrium, then the last market must also be in equilibrium, a property which is essential in the proof of the existence of equilibrium. It follows from the statement of the Walra Law that the market value of total excess demand in the economy must be zero.

5.5 Existence, Uniqueness and Stability of General Equilibrium

Three problems arise in connection with a general equilibrium:

1. Does a general equilibrium solution exist? (Existence problem.)
2. If an equilibrium solution exists, is it unique? (Uniqueness problem.)
3. If an equilibrium solution exists, is it stable? (Stability problem.)

These problems can best be illustrated with the partial-equilibrium example of a demand-supply model. Assume that a commodity is sold in a perfectly competitive market, so that from the utility-maximising behaviour of individual consumers there is a market demand function, and from the profit-maximising behaviour of firms, we have a market supply function. An equilibrium exists when at a certain positive price the quantity demanded is equal to the quantity supplied.

The price at which Qd = Qs is the equilibrium price. At such a price there is neither excess demand nor excess supply. (The latter is often called negative excess demand.) Thus an equilibrium price can be defined as the price at which the excess demand is zero the market is cleared and there is no excess demand.

The equilibrium is stable if the demand function cuts the supply function from above. In this case an excess demand drives price up, while an excess supply (excess negative demand) drives the price down (figure 5.1).

The equilibrium is unstable if the demand function cuts the supply function from below. In this case an excess demand drives the price down, and an excess supply drives the price up (figure 5.2).

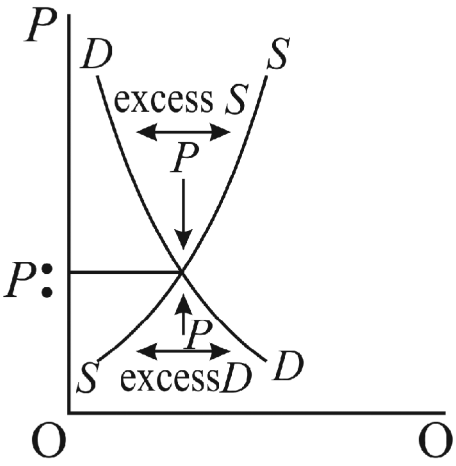


Figure 5.1 : Unique, Stable equilibrium

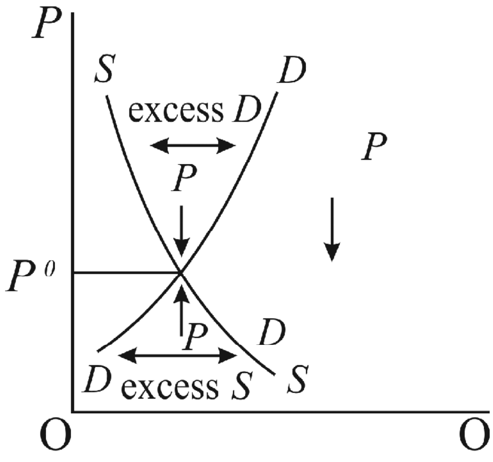


Figure 5.2 : Unique, Unstable equilibrium

In figure 5.3, we depict the case of multiple equilibria. It is obvious that at P^e_1 there is a stable equilibrium, while at P^e_2 the equilibrium is unstable. Finally, in figure 5.4 an equilibrium (at a positive price) does not exist.

It should be clear from the above discussion that,

(a) the existence of equilibrium is related to the problem of whether the consumers' and producers' behaviour ensures that the demand and supply curves intersect (at a positive price);

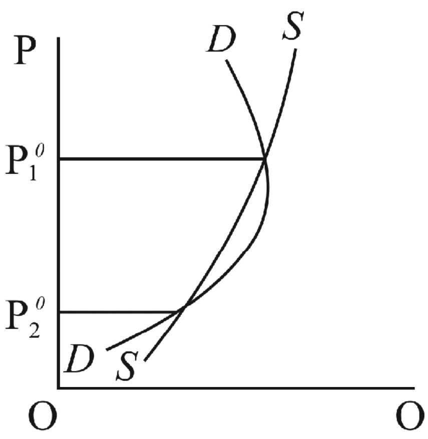


Figure 5.3 : Mutiple Equilibria

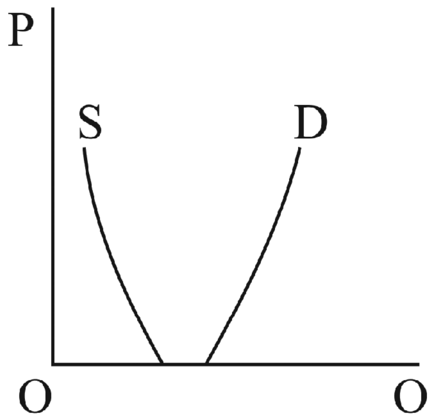


Figure 5.4 : No Equilibrium Exists

(b) the stability of equilibrium depends on the relationship between the slopes of the demand and supply curves;

(c) the uniqueness of equilibrium is related to the slope of the excess demand function, that is, the curve which shows the difference between Q_D and Q_s at any one price.

In fact, the three basic questions related to the problem of existence, stability and uniqueness of an equilibrium can be expressed in terms of the excess demand function.

$$E(_{Pi}) = Q_D(_{Pi}) — Qs(_{Pi})$$

To see this, we redraw below figures 5.5 – figure 5.8 in terms of the excess demand function. For each of these cases i.e. from figure 5.1 - figure 5.4, we have derived the relevant excess demand function by subtracting Q_s from Q_D at all prices.

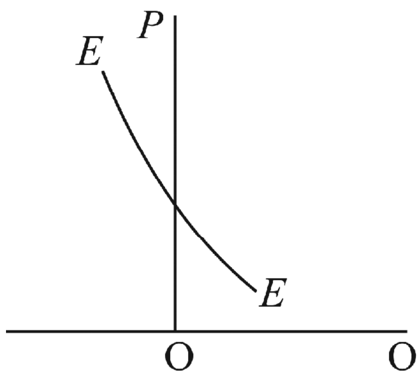


Figure 5.5 : Stable Equilibrium, $E_{(P)} < 0$

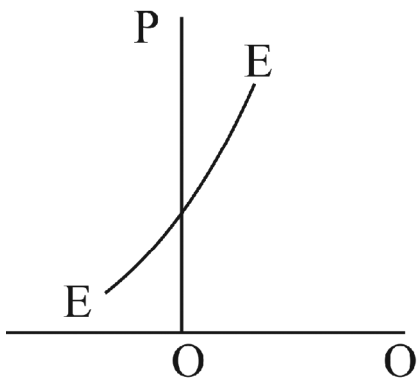


Figure 5.6 : Stable Equilibrium, $E_{(P)} > 0$

From the redrawn diagrams (in conjunction with the corresponding ones figure 5.1 – figure 5.4) we can draw the following conclusions:

1. The excess demand function, $E_{\{P\}}$, intersects the vertical (price)-axis when there is an equilibrium, that is, when the excess demand is zero. If $Q_D = Q_s$, then $E_{(P)} = 0$.

2. There are as many equilibria as the number of times that the excess demand curve $E_{(P)}$ intersects the vertical price-axis (figure 5.7).

3. The equilibrium is stable if the slope of the excess demand curve is negative at the point of its intersection with the price-axis (figure 5.5).

4. The equilibrium is unstable if the slope of the excess demand curve is positive at the point of its intersection with the price-axis (figure 5.6).

5. If the excess demand function does not intersect the vertical axis at any one price, an equilibrium does not exist (figure 5.8).

The above analysis of the existence, stability and uniqueness in terms of excess demand functions has been extended to general equilibrium analysis under the 2 x 2 x 2 model.

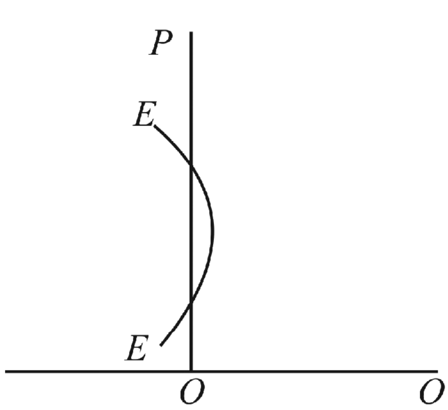


Figure 5.7 : Multiple Equilibria

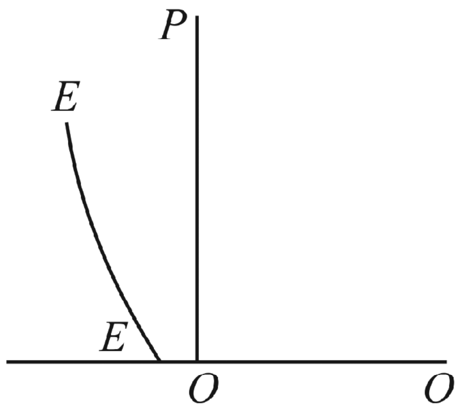


Figure5. 8 : No Equilibrium exists

We have shown above that under certain conditions, an equilibrium price, can be found, for which demand is equal to supply. However, the underlying process through which the initial price P moves to equilibrium P* is seen through various mechanisms. That is to say, various mechanisms for reaching the Walrasian Equilibrium(WE) have been proposed. Here are the following:

i) **Tatonnemant Process :** This process is the most famous. A fictitious auctioneer calls out a price vector and economic agents respond by providing information to the auctioneer about their demands and supplies at these prices. If some markets do not clear, the auctioneer proposes a new set of prices, and the process is repeated. This goes on until equilibrium is not reached.

ii) **Provisional Contract :** There could be recontracting in which buyers and sellers would enter into provisional contract before actual exchange goods takes place. Unless the agreed price vector is equilibrium one, provisional contracts remain unimplemented.

iii) **Central Planning :** A central authority is established to whom the consumers report their excess demands at all prices. This was the mechanism suggested by Oskar Lange and Abba P. Lerner to show that that a socialist system can in principle operate as efficiently as a capitalist one.

iv) **Market makers:** We can also imagine the existence of a market maker. An individual or a number of individuals can set up stalls to buy and sell so that others do not have to wander around in search of trading opportunities.

Problems with the concept of Walrasian Equilibrium (WE):

There are a number of features of a WE that we ought to worry about:

- The existence of a WE requires that each consumer be aware of all the prices. But in an uncertain world, today's choices depend on forecasts of tomorrow's prices and trading opportunities. Consumers cannot correctly forecast these.
- Consumers may not act rationally, i.e., try to maximize their utilities subject to the budget constraints. We may question whether utility functions exist at all or whether consumers have utility maximization as their goal.
- Consumers must be able to buy as much as or as little they want to at the going prices, but consumers might want to transact such large quantities that they affect prices. Also in many economies, commodities are sometimes in excess demand and rationing is not unknown.

5.6 Two-sector General Equilibrium Models

Now we turn to analyze the general equilibrium of a simple economy in which there are two factors of production, two commodities (each produced by a firm) and two consumers. This is known as the 2 x 2 x 2 general equilibrium model. We will restrict our analysis to the perfectly competitive market system; since with free competition it has been proved that a general equilib-rium solution exists (given some additional assumptions about the form of the pro-duction and demand functions).

Assumptions of the 2 X 2 X 2 Model :

1. There are two factors of production, labour (L) and capital (K), whose quanti-ties are given exogenously. These factors are homogeneous and perfectly divisible.
2. Only two commodities are produced, X and Y. Technology is given. The pro-duction functions of the two commodities are represented by two iso-quant maps, with the usual properties. The isoquants are smooth and convex to the origin, imply-ing diminishing marginal rate of factor (technical) substitution along

any iso-quant. Each production function exhibits constant returns to scale. Finally, it is assumed that the two production functions are independent: there are no external economies or diseconomies for the production activity of one product arising from the produc-tion of the other.

3. There are two consumers in the economy, A and B, whose preferences are represented by ordinal indifference curves, which are convex to the origin, exhibiting diminishing marginal rate of substitution between the two commodities. It is assumed that consumer choices are independent: the consumption patterns of A do not affect B's utility, and vice versa. Bandwagon, snob, Veblenesque and other 'exter-nal' effects are ruled out. Finally, it is assumed that the consumers are sovereign, in the sense that their choice is not influenced by advertising or other activities of the firms.

4. The goal of each consumer is the maximisation of his own satisfaction (utility), subject to his income constraint.

5. The goal of each firm is profit maximisation, subject to the technological con-straint of the production function.

6. The factors of production are owned by the consumers.

7. There is full employment of the factors of production, and all incomes received by their owners (A and B) are spent.

8. There is perfect competition in the commodity and factor markets. Consumers and firms pursue their goals faced by the same set of prices (P_x, P_y, w, r).

In this model a general equilibrium is reached when the four markets (two commo-dity markets and two factor markets) are cleared at a set of equilibrium prices (P_x, P_y, w, r) and each participant economic agent (two firms and two consumers) is simultaneously in equilibrium.

The general equilibrium solution thus requires the determination of the values of the following variables:

The total quantities of the two commodities X and Y, which will be produced by firms and bought by the consumers.

The allocation of the given K and L to the production of each commodity (K_x, K_y, L_x, L_y).

The quantities of X and Y which will be bought by the two consumers (X_A, X_B, Y_A, Y_B).

The prices of commodities (P_x and P_y) and of the factors of production (wage w, and rental of capital r).

In figure 5.9, the general equilibrium solution is shown by points T (on the production possibility curve) and T (on the Edgeworth contract curve). These points define six of the 'unknowns' of the system, namely the quantities to be produced of the two commodities (X_e and Y_e), and their distribution among the two consumers ($X^A{}_e$, $X^B{}_e$, $Y^A{}_e$, $Y^B{}_e$). We examine the determination of the allocation of resources between X and Y. The determination of the remaining unknowns (prices of factors and commodities is examined in separate sections below.

Point T on the production transformation curve (figure 9) defines the equilib-rium product mix Y_e and X_e. Recalling that the PPC is the locus of points of the Edgeworth contract curve of production mapped on the product space, point T corresponds to a given point on this contract curve, say T´´ in figure 9. Thus T´´ defines the allocation of the given resource endowments in the production of the general equilibrium commodity mix. The production of X_e absorbs L_x of labour and K_x of capital, while Y_e employs the remaining quantities of factors of production; L_y and K_y. Thus four more 'unknowns' have been defined from the general equilibrium solution.

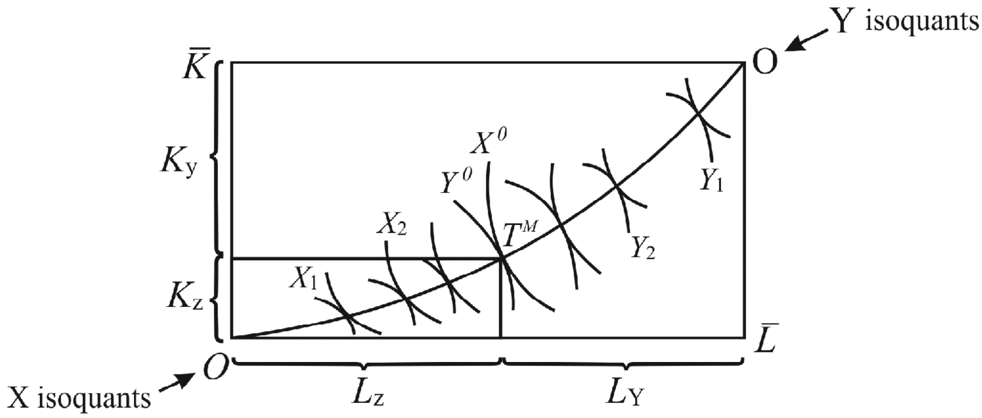


Figure 5.9 : Factor allocations between X and Y

Prices of commodities and factors:

The next step in our analysis is to show the determination of prices in the general equilibrium model, under perfect competition.

In the simple 2 x 2 x 2 model there are four prices to be determined, two commo-dity prices, P_x and P_y, and two factor prices, the wage rate w, and the rental of capital r. We thus need four independent equations. However, given the assumptions of the simple model, we can derive only three independent relations.

1. Profit maximisation by the individual firm implies least-cost production of the profit-maximising output. This requires that the producer adjusts his factor mix until the MRTS of labour for capital equals the w/r ratio,

$$MRTS^x_{L,k} = w/r = MRTS^y_{L,K} \quad (1)$$

In other words the individual producer maximises his profit at points of tangency between the isoquants and isocost lines whose slope equals the factor price ratio.

2. In perfect factor and output markets the individual profit-maximising producer will employ each factor up to the point where its marginal physical product times the price of the output it produces just equals the price of the factor

$$w = (MPP_{L,X}).(P_X) = (MPP_{L,y}).(P_y) \quad (2)$$

$$r = (MPP_{K,x}).(P_x) = (MPP_{k,y}).(P_y) \quad (3)$$

3. The individual consumer maximises his utility by purchasing the output mix which puts him on the highest indifference curve, given his income constraint. In other words maximisation of utility if attained when the budget line, whose slope is equal to the ratio of commodity prices P_x/P_y, is tangent to the highest utility curve, whose slope is the marginal rate of substitution of the two commodities

$$MRS^A_{y,X} = P_y/P_x = MRS^B_{y,x} \quad (4)$$

Although we have four relations between the four prices, one of them is not indepen-dent. Because, dividing (2) and (3), we obtain (5) as

$$\frac{w}{r} = \frac{MPP_{L,x}}{MPP_{K,x}} = \frac{MPP_{L,y}}{MPP_{K,y}} = (MRTS_{L,K}) \quad (5)$$

which is the same as expression (4). Thus we have three independent equations in four unknowns. Apparently the absolute values of w, r, P_x and P_y are not uniquely determined (although the general equilibrium solution is unique). Prices in the Walrasian system are determined only up to a ratio or a scale factor. We can express any three prices in terms of the fourth, which we choose arbitrarily as a numeraire or unit of account. For example assume that we choose P_x as the numeraire.

The remaining three prices can be determined in terms of P_x as follows :

From equation (5) we obtain,

$$w = r(MRTS_{L,k})$$

From the first part of equation (5) we have

$$r = (MPP_{K,x})P_x$$

Substituting r and w we obtain,

$$w = (MRTS_{L,K})(MPP_{K,x})P_x$$

Finally, we obtain

$$P_y = (MRS_{x,y})P_x$$

Given the relative prices w, r and P_y, that is, the prices relative to numeraire P_x :

$$\frac{P_y}{P_x} = (MRS_{x,y})$$

$$\frac{W}{P_x} = (MRTS_{L,K})\,(MPP_{K,x})$$

$$\frac{r}{P_x} = (MPP_{K,x})$$

The terms in brackets are known values, that is, values determined by the general equilibrium solution and the maximising behaviour of economic decision-makers with a given state of technology and given tastes.

Note that any good can serve as numeraire, and the change of numeraire leaves the relative prices unaffected. We can also assign any numerical value to the price of the numeraire. For convenience P_x is assigned the value of 1. But if, for example, we choose to set P_x = £b, then the price of y in £ will be **$P_y = b.\ P_y/P_x$ (pounds)**

This, however, does not mean that the absolute level of the prices of the system is determined. It simply illustrates the fact that we can assign to the price of the numeraire any value we choose. However, the general equilibrium model can be completed by adding one (or more) monetary equation. Then the absolute values of the four prices can be determined. Unless a market for money is explicitly in-troduced, the price side of the model depends on an endogenous numeraire.

5.7 Compensation Principle

Compensation principle is the rule that redistribution leads to an improvement in economic welfare if those who gain an increase in real income and welfare are able to compensate the losers and still be better off. The major principle of modern welfare economics was devised by Kaldor and Hicks to deal with the problem of making interpersonal comparison of utility. This principle is often been applied in cost-benefit analysis.

5.7.1 Kaldor-Hicks Scitovsky tests

Kaldor, Hicks and Scitovsky have given their tests for judging an increasing in welfare. Like Pareto, they isolate the problem of production from that of distribution. They deal with policy change with ambiguous welfare effects saying that if the people benefiting from it can gainfully compensate the losers, then the policy change is desirable otherwise it is not. This extension is popularly called the compensation principle. The discussion follows in the subsequent sections.

5.7.2 Kaldor-Hicks criterion in terms of utility possibility curve

According to Kaldor, the test of increase in social welfare is that if some people are made better off and others worse off, the gainers from the change could more than compensate the losers, and yet be better off them. The actual payment of compensation is regarded as a political or ethical decision. Kaldor does not require that the losers should actually be compensated. Rather, he requires that the gainers should be able potentially to compensate the losers out of their gains.

Hicks presents the same criterion in a little different way thus: “If A is made so much better off by the change that he could compensate B for his loss, and still have something left over, then the reorganization is unequivocal improvement.”

Thus the Kaldor-Hicks criterion implies that if an economic change leads to the production of more goods and services they can be so distributed as to make some people better off and none worse off. Actual redistribution, being a political or ethical issue, need not take place. It is enough that reorganization creates such conditions that redistribution can be effected.

5.7.3 Criticism of Kaldor-Hicks in terms of community indifference curve

1. Ignores income distribution: The Kaldor-Hicks compensation principle, according to Dr. Little, is merely a definition and not a 'test' of increase in welfare because it ignores income distribution.

2. Measures only potential welfare: In trying to separate production from distribution, this criterion confuses potential welfare with actual welfare. It simply measures potential welfare changes associated with changes in any particular bundle of goods. The actual welfare depends not only on the production of goods and services but also on their distribution which is ignored here.

3. Not free form Interpersonal Comparisons: Kaldor, Hicks and their followers failed in their efforts to find out a value-free criterion. The Kaldor-Hicks criterion is based on the assumption that the 'social value of money' is the same in the hands of both the rich and the poor. Moreover, money is not actually transferred but remains with the better off.

4. Based on Long-Run Welfare Adjustments: Little and Scitovsky have criticised Hicks for suggesting long-run welfare adjustments which would have insignificant real income distribution effects. Moreover, the effects would be random and negligible so that they cancel out in the long-run. If, however, the time period is long enough, even the people who are better off would be dead and this criterion then becomes meaningless is where the critique lies.

5. Does not Involve Actual Compensation: This criterion does not take into consideration the payment of actual compensation. It recognises only potential compensation with which actual increase in welfare cannot be measured. Therefore, actual compensation is necessary so that no individual remains a loser. The problem, however, is the payment of actual compensation may involve many administrative problems which render this criterion impracticable.

6. No Universal Validity: Scitovsky has criticised Kaldor for the view that the state is fully responsible for maintaining an equitable distribution of income and correcting it if need be. But then it means, in case of the market dominated economies this principle does not work.

5.7.4 Scitovsky double criterion

To rule out the possibility of contradictory results in Kaldor-Hicks criterion Scitovsky formu-lated a "double criterion" approach which requires the fulfillment of Kaldor-Hicks criterion and also the fulfillment of the *reversal test*. It means that a change is an improvement if the gainers in the changed situa-tion are able to persuade the losers to accept the change and simultaneously losers are not able to persuade the gainers to remain in the original situ-ation.

5.7.5 Samuelson's criterion

The concept of social welfare function was first introduced by Prof. Bergson)as discussed in the next section in details) and later on developed by Samuelson, Tintner and Arrow. They are of the opinion that no meaningful propositions in welfare economics can be made without introducing value judgments. The concept of social welfare is an attempt at providing a scientifically normative study of welfare economics. Bergson defines it "as a function either of the welfare of each member of the community or of the quantities of products consumed and services rendered by each member of the community." In its original form the Bergson social welfare function is formulated in a completely general manner. It is a function which establishes a relation between social welfare and all possible variables which affect each individual's welfare, such as a services and consumption of each individual. It is an ordinal index of society's welfare and is a function of individual utilities. It is expressed as,

$$W = F(U_1,.U_2, \ldots U_n)$$

where W is the social economic welfare, F is for function, and $U_r U_2 \ldots U_N$ is the levels of utilities of 1, 2,…individuals. W is an increasing function of these utilities.

The general properties of the social welfare function are similar to those of an individual utility function. In particular, the value of the welfare index increases whenever the utility level of one individual is increased without lowering that of the other individual. Thus the social welfare function is consistent with the Pareto optimality criterion, but it goes much further, since it assigns a value to every economic state, including those which according to the Pareto criterion are regarded as non-comparable. The existence of a social welfare function, therefore, implies a comparison of the welfare position of the individual members of society.

5.7.6 Little's criterion

Dr. Little has developed a reaction against the compensation criteria proposed by Kaldor, Hicks and Scitovsky. In form, it is also a compensation criterion, but in spirit, it differs markedly from the earlier Kaldor-type criteria. Dr. Little asserts that neither the Kaldor-Hicks test nor the Scitovsky double test, either alone or together, can possibly be taken as a criterion of welfare. Since Little believes that value judgements are essential in welfare economics, he bases his criterion on two value premises. 1. The well-being of an individual is supposed to be greater in a chosen position that it is in any other position. 2. Any social alternation that makes everybody better off is a good thing. Based on these value judgements, the criterion can be stated in this way: An economic change constitutes social improvement (a) if the resulting redistribution is no worse than the old and (b) if it is impossible to make the community as well off in the initial position as it would be after the change.

5.7.7 Amartya Sen's Approach

The Capability Approach is defined by its choice of focus upon the moral significance of individuals' capability of achieving the kind of lives they have reason to value and thus define their social welfare. This distinguishes it from established approaches to ethical evaluation, such as utilitarianism or resourcism, which focuses exclusively on subjective well-being. A person's capability to live a good life is defined in terms of the set of valuable 'beings and doings' like being in good health, access to education or having loving relationships with others to which they have real access. Here 'poverty' is understood as deprivation in the capability to live a good life, and 'development' is understood as capability expansion.

5.7.8 Fundamental Welfare Theorem

Before going to the discussion on social welfare functions, the two Fundamental Welfare Theorems of Economics can be defined as :

A. First Fundamental Welfare Theorem

Every competitive equilibrium is Pareto efficient. That is to say a 'free market' in equilibrium is Pareto efficient, provided the following conditions true :

1) No externalities
2) Perfect competition
3) No transaction costs
4) Full information

Assume that all individuals and firms are self-interested price takers. Then a competitive equilibrium is Pareto optimal.

To illustrate the theorem, we focus on one simple version of it, set in a pure production economy. For a general versions of the theorem, with both production and exchange, the reader can refer to Mas-Colell, Whinston & Green (1995).

In a general equilibrium production economy model, there are K firms and m goods, but, for simplicity, no consumers. We write $k = 1, 2, ..., K$ for the firms, and

$j = 1, 2, ..., m$ for the goods. Given a list of market prices, each firm chooses a feasible

input–output vector y_k so as to maximize its profits. We adopt the usual sign convention for a firm's input–output vector y_k: $y_{kj} < 0$ means firm k is a net *user* of good j, and $y_{kj} > 0$ means firm k is a net *producer* of good j. When we add the amounts of good j over all the firms, $y_{1j} + y_{2j} + \ldots + y_{Kj}$, we get the aggregate net amount of good j produced in the economy, if positive, and an aggregate net amount of good j used, if negative. What is feasible for firm k is defined by some fixed production possibility set Y_k. Under the sign convention on the input–output vector, if p is a vector of prices, firm k's profits are given by

$$\pi_k = p.y_k.$$

A list of feasible input–output vectors $y = (y_1, y_2, \ldots y_K)$ is called a *production plan* for the economy. A *competitive equilibrium* is a production plan $\hat{y}$ and a price vector p such that, for every k, $\hat{y}k$ maximizes p_k subject to y_k's being feasible. (Since the production model abstracts from the ultimate consumers of outputs and providers of inputs, the supply equals demand requirement for an equilibrium is moot).

If $y = (y_1, y_2, ¼, y_K)$ and $z = (z_1, z_2, ¼, z_K)$ are alternative production plans for the economy, z is said to *dominate* y if the following vector inequality holds:

$$\sum_k z_k \geq \sum_k y_k,$$

The production plan y is said to be *Pareto optimal* if there is other production plan that dominates it. (Note that for two vectors a and b, $a \geq b$ for every good j,

means $a_j \geq b_j$

with the strict inequality holding for at least one good.)

We now have the apparatus to state and prove the First Theorem in the context of the pure production model:

Assume

that all prices are positive, and that $\hat{y}$, p is a competitive equilibrium. Then $\hat{y}$ is Pareto optimal.

To see why, suppose to the contrary that a competitive equilibrium production plan $\hat{y} = (\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_K)$ is not optimal. Then there exists a production plan $z = (z_1, z_2, \ldots, z_K)$ that dominates it. Therefore

$$\sum_k z_k \geq \sum_k \hat{y}_k .$$

Taking the dot product of both sides with the positive price vector p gives

$$\rho \cdot \sum_k z_k > \sum_k \hat{y}_k .$$

But this implies that, for at least one firm k,

$$\rho \cdot z_k > \rho \cdot \hat{y}_k,$$

which contradicts the assumption that $\hat{y}k$ maximizes firm k's profits

The First Theorem of Welfare Economics is mathematically true but nevertheless open to objections. Here are the commonest: (1) The theorem is an abstraction that ignores the facts. Preferences of consumers are not given, they are created by advertising. The real economy is never in equilibrium, most markets are characterized by excess supply or excess demand, and are in a constant state of flux.

The economy in dynamic, tastes and technology are constantly changing, whereas the model assumes they are fixed. The cast of characters in the real economy is constantly changing, the model assumes it fixed.

(2) The theorem assumes competitive behaviour, whereas the real world is full of monopoly and market power.

(3) The theorem assumes there are no externalities. In fact, if in an exchange economy person l's utility depends on person 2's consumption as well as his own, the theorem does not hold. Similarly, if in a production economy firm *k*'s production possibility set depends on the production vector of some other firm, the theorem breaks down. In a similar vein, the theorem assumes there are no public goods, that is, goods like national defense, judicial systems, or lighthouses, that are necessarily non-exclusive in use. If such goods are privately provided (as they would be in a completely *laissez-faire* economy), then their level of production will be sub-optimal.

(4) The theorem ignores distribution. *Laissez-faire* may produce a Pareto optimal outcome, but there are many different Pareto optima, and some are fairer than others. Some people are endowed with resources that make them extremely rich, while others, through no fault of their own, are extremely poor.

The first and second objections to the First Theorem are beyond the scope of this entry. The third, regarding externalities and public goods, is one that economists have always acknowledged. The standard remedies for these market failures involve various modifications of the market mechanism, including Pigovian taxes (Pigou, 1920) on harmful externalities, or appropriate Coasian legal entitlements to, for example, clean air (Coase, 1960).

The important contribution of Pigou is set in a partial equilibrium framework, in which the costs and benefits of a negative externality can be measured in money terms. Suppose that a factory produces gadgets to sell at some market-determined price, and suppose that, as part of its production process, the factory emits smoke which damages another factory located downwind. In order to maximize its profits, the upwind factory will expand its output until its marginal cost equals price. But each additional gadget it produces causes harm to the downwind factory – the marginal external cost of its activity. If the factory manager ignores that marginal external cost, he will create a situation that is non-optimal in the sense that the aggregate net value of both firms' production decisions will not be as great as it could be. That is, what Pigou calls 'social net product' will not be maximized, although 'trade net product' for the polluting firm will be. Pigou's remedy was for the state to eliminate the divergence between trade and social net product by imposing appropriate taxes (or, in the case of beneficial externalities, bounties). The Pigovian tax would be set equal to marginal external cost, and with it in place the gap between the polluting firm's view of cost and society's view would be closed. Optimality would be re-established.

Coase's contribution was to emphasize the reciprocal nature of externalities and to suggest remedies based on common law doctrines. In his view the polluter damages the pollutee only because of their proximity, e.g., the smoking factory harms the other only if it happens to locate close downwind. Coase rejects the notion that the state must step in and tax the polluter. The common law of nuisance can be used instead. If the law provides a clear right for the upwind factory to emit smoke, the downwind factory can contract with the upwind factory to reduce its output, and if there are no impediments to bargaining, the two firms acting together will negotiate an optimal outcome.

Alternatively, if the law establishes a clear right for the downwind factory to recover for smoke damages, it will collect external costs from the polluter, and thereby motivate the polluter to reduce its output to the optimal level. In short, a legal system that grants clear rights to the air to either the polluter or pollutee will set the stage for an optimal outcome, provided that bargaining is costless. If bargaining is costly, then the law should be designed with an eye towards minimizing social costs created by the externality.

With respect to public goods, since Samuelson (1954) derived formal optimality conditions for their provision, the issue has received much attention from economists; one especially notable theoretical question has to do with discovering the strengths of people's preferences for a public good. If the government supplies a public judicial system, for instance, how much should it spend on it (and tax for it)? At least since Samuelson, it has been known that financing schemes like those proposed by Lindahl (1919), where an individual's tax is set equal to his marginal benefit, provide perverse incentives for people to misrepresent their preferences. Schemes that are immune to such misrepresentations (in certain circumstances) have been developed (Clarke, 1971; Groves and Loeb, 1975).

But it is the fourth objection to the First Theorem that may be most fundamental.

What about distribution?

There are two polar approaches to rectifying the distributional inequities of *laissez-faire*. The first is the *command economy* approach: a central bureaucracy makes detailed decisions about the consumption decisions of all individuals and production decisions of all producers. The main theoretical problem with the command approach is that it fails to create appropriate incentives for individuals and firms. On the empirical side, the experience of the late Soviet and Maoist command economies establish that highly centralized economic decision making leaves much to be desired, to put it mildly.

The second polar approach to solving distribution problems is to transfer income or purchasing power among individuals, and then to let the market work. The only kind of purchasing power transfer that does not cause incentive-related losses is the lump-sum money transfer. Enter at this point the standard remedy for distribution problems, as put forward by market-oriented economists, and our second major theorem.

The Second Fundamental Theorem of Welfare Economics establishes that the market mechanism, modified by the addition of lump-sum transfers, can achieve virtually *any* desired optimal distribution. Under more stringent conditions than are necessary for the First Theorem, including assumptions regarding quasi-concavity of utility functions and convexity of production possibility sets, the Second Theorem gives the following:

B. Second Fundamental Welfare Theorem

If all individuals have indifference curves that are strictly convex to the origin, then any Pareto optimal state of the economy that assigns positive quantities of every good to every person can be achieved as a competitive equilibrium for an appropriate reallocation of initial endowment given a price vector.

Assume that all individuals and producers are self-interested price takers. Then almost any Pareto optimal equilibrium can be achieved via the competitive mechanism, provided appropriate lump-sum taxes and transfers are imposed on individuals and firms.

C. Third Fundamental Theorem of Welfare Economics:

There is no Arrow Social welfare function that satisfies the conditions of universality, Pareto consistency, Independence, non-dictatorship.

5.8 Social Welfare Function

5.8.1 Bergson Frontier

The concept of ‘Social Welfare Function’ was propounded by A. Bergson in his article ‘A Re-formulation of Certain Aspects of Welfare Economics’ in 1938. Prior to its various concepts of social welfare had been given by different welfare theorists but they failed to provide a satisfactory solu-tion to the problem of maximization of social welfare and measurement. By providing the concept of social welfare function Bergson and later on, additions by Samuelson have attempted to provide a new approach to the domain welfare economics and have succeeded in rehabilitating welfare econom-ics. They have put forward the concept of social welfare function that considers only the ordinal preferences of individuals. Social welfare function is an ordinal index of society’s welfare and is a function of the utility levies of all individuals constituting the society. Bergson-Samuelson social welfare function can be written in the following manner:

$$W = W (U_1, U_2, U3…………., U_n)$$

where, W If represents the social welfare U_1, U_2, U_3, ..., U_n represent the ordinal utility indices of different individuals of the society.

Value judgements determine a form of the social welfare function; with a different set of value judgements, the form of social welfare function would be different. We can explain the social welfare function with the help of social indifference curves or welfare frontiers. Let us assume a society of two persons. In such a case social welfare function can be represented with the help of social indifference curves.

The utilities of individuals A and B have been represented on the horizontal and vertical axes respectively. W_1, W_2 and W_3 are the social indifference curves representing succes-sively higher levels of social welfare. A social indifference curve is a locus of the various combinations of utilities of A and B which result in an equal level of social welfare. The properties of social indifference curves are just like that of individual consumer’s indifference curves (convex to the origin, higher social indifference curves mean higher welfare, etc.).

By including the concept of grand utility possibility frontier along with Bergson-Samuleson social welfare function we are able to obtain a unique optimum position or maximum social welfare position which is explained below. As shall be explained below, a grand utility possibility frontier is a locus of the various physi-cally attainable utility combinations of two persons when the factor endowments, state of technology and preference orders of the individuals are given. In other words, every point on the grand utility possibility curve represents the optimum position with regard to the allocation of the products among the consumers, allocation of factors among different products and the direction of production. Thus every point on the grand utility possibility curve represents a Pareto optimum and as we move from one point to another on it the utility of one individual increases while that of the other falls.

Now, let us superimpose grant utility possibility curve on the social indifference curves repre-senting social welfare function to find a unique optimum position of social welfare. In figure 10 social indifference curves W_1, W_2, W_3 and W_4 representing the social welfare function have been drawn along with the grand utility possi-bility curve VV'.

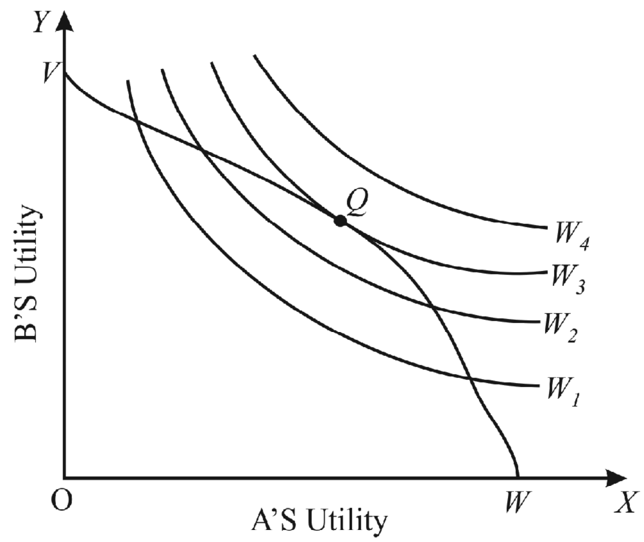


Figure 5.10 : Social Welfare Function and Point of Constrained Bliss

Social indifference curve W_3 is tangent to the grand utility possibility curve VV' at point Q. Thus, point Q repre-sents the maximum possible social welfare given the factor endowments, state of technol-ogy and preference scales of the individuals. Point Q is known as the point of constrained bliss since, given the constraints regarding factor endowments and the state of technol-ogy.

Q is the highest possible state of social welfare which the society can attain. Social welfare represented by the social indifference curve W_4 is higher than social indifference curve W_3 passing through Q but it is not pos-sible to attain it, given the technology and fac-tor endowment.

Thus, from among a large number of Pareto optimum points on the grand utility possibility curve, we have a unique optimum point Q at which the social welfare is the maximum.

5.8.2 Arrow's Social Choice and Individual Values

Prof. Kenneth Arrow, in his monumental work, *Social Choice and Individual Values*, published in 1951, shows that the task of constructing a social welfare functions to reflect the aims and aspirations of a free democratic society is an impossible one. Arrow has proved a general theorem about the impossibility of constructing an ordering for society as a whole which will in some way reflect all the individual orderings of the members who make up the society.

The first condition may be called *universality condition*. It states that a definite social ordering is derivable from a reasonably wide range of individual orderings. This social ordering must have the properties of transitivity. By the axiom of transitivity, any two alternatives must be related either by preference or indifference. Thus, for any two alternatives X and Y, either X is preferred to Y, on Y is preferred to X or the two are indifferent. By the axiom of transitivity if X is preferred on indifferent to Y and Y is preferred on indifferent to Z, then X must be either preferred on indifferent to Z. These two axioms constitute the foundation of modern choice theory.

The second condition is called *Responsiveness condition*. It states that social ordering is positively related with the individual orderings. On simply social choices must move in the same direction as individual choices.

The third condition is called by Arrow *the Independence of Irrelevant Alternatives.* It simply, states that the choice made by a society depends on the orderings of individuals in that environment and not on the orderings of alternatives outside that environment.

Fourth is the *non-imposition condition* also called the *conditions of citizens sovereignty.* It required that there should be no external control over a society's choice. The social welfare function is not to be imposed.

Condition number 5, called, the *condition of Non-dictatorship* is a part of the condition 4. It permits the construction of social choices by collective methods and not by dictatorial ones. Hence the condition is the social welfare function is not to be dictatorial.

Condition 1 specifies the scope of social welfare function and other four are value judgements. Arrow next considers whether a social ordering can be derived from any set of individual orderings. He demonstrates that impossibility of doing this without violating at-least one of the value judgements as expressed in five conditions. This is his "General Possibility Theorem". Arrow first considers a simple case where there are only two alternatives and shows that in this case the method of majority decision yields a social welfare function satisfying all the five conditions.

But when there are three or more alternatives difficulty emerges and no valued social welfare function can be derived therefore a social welfare function may be either imposed on dictatorial. Arrows offers three important deductions-consequences 1,2 and 3. The three alternatives are X,Y,Z and there are two individuals. Consequence I states that whenever both individuals prefer X to Y, society will prefer X to Y. Consequence 2 states that, if in four given choice the will of individual 1 prevails against the opposition of 2, then individual 1 prevails against the opposition of 2, then individual 1's view will prevail if 2 is indifferent on if he agrees with 1. Consequence III states that, if two individuals have opposing interests, then the society will be indifferent between the two alternatives.

Based on these consequences, the General Possibility Theorem is stated in its simplest form. Let there be two individuals and three alternatives X,Y,A. If individual 1 prefers X to Y and individual 2, Y to X, the society is indifferent between the two. If individual 1 has X,Y,Z and individual 2 has the ordering Z,X,Y. Since individual one prefers Y to Z and individual 2 prefers Z to Y, the society should be indifferent between the two. For both X is preferred to Y and society also prefers X to Y. By the axiom of transitivity society prefers X to Z.

Since individual 1 prefers X to Z and 2 prefers Z to X we are to concluded that the society is indifferent as between X and Z. But this contradicts the earlier conclusion that X is preferred to Z. It can't be that for the society, X is both preferred and also indifferent to Z. This is the idea of Arrow's Impossibility Theorem.

Criticism:

Arrows insights have not gone without criticism. His impossibility theorem in particular has sparked a literature that has found other impossibilities as well as some possibility results. For example, Amartya Sen (1982) has suggested at least two other alternatives based on relaxation of transitivity and removal of the Pareto principle. This has enabled

Sen to show the existence of voting mechanisms that comply with all of Arrow's criteria but supply only semi- transitive results.

5.9 Conclusion

In this unit, we have discussed how the non-feasibility of Pareto Optimum gives rise to the next best alternative, the second best and analyzed some alternative theories such as Arrow's impossibility theorem. Summing up, this unit focuses on the compensation principle as an improvement over the Paretian optimality. We have also discussed the two different ways of Kaldor and Hicks exposition of Pareto optimality from the gainers and losers' point of view respectively. Further, it is explained how Hicks criterion revert the Kaldor's criterion. Then we examined the Scitovsky reversal criteria and how Samuelson's criterion offers a solution. Finally, we end with a note on the social welfare functions.

5.10 Summary

Arrow's Impossibility Theorem : Any social decision rule which satisfies the requirements of (1) completeness, reflexivity and transitivity (to be able to compare social preferences), (2) universality of application, (3) Pareto consistency, (4) independence of irrelevant alternatives (if people's feelings change about some set of irrelevant allocations, but do not change about the pair of allocations x and y, the social preference rule must preserve the social ordering of x and y) makes one a dictator. There can be rule which satisfies all the five requirements.

The significance of the Arrow theorem: Despite negative nature of Arrow's conclusion, it should be remembered that all societies do in fact make choices. India and China adopt five year plans ; and Alaskan Eskimos decide how to improve upon their communal fishing methods next year.

Bergson social welfare function: In 1938, Abram Bergson (1914— 2003) created the new welfare economics by asserting that a social welfare function can be established by attaching weights to each individual's welfare function: this rejected the earlier cardinal utility approach. Bergson, in his approach, intended to challenge the view that a community's welfare is the sum of individuals' welfare.

Coase Theorem: It states that when private parties can bargain in the absence of transaction costs, externality issues may be resolved without the need of government intervention.

Contract Curve: The set of all the efficient allocations of goods among those individuals in an exchange economy. Each of these allocations has the property that no one individual can be made better off without making someone else worse off as shown in figure 9.

Convexity Assumptions : Assumptions about the shapes of individuals' utility functions and firms' production functions.It is important because they ensure that the application of first-order conditions will indeed yield a true maximum.

Edgeworth Box diagram: A graphic device used to demonstrate economic efficiency It is most frequently used to illustrate the contact curve in an exchange economy.

Equilibrium prices set in a Walrasian equilibrium: The price in which proves the existence of excess supply in one market must be matched by excess demand in another market so it balances out. Mathematically

$$\sum_{j=1}^{k} p_j \cdot D_j - \sum_{j=1}^{k} p_j \cdot S_j = 0$$

where p_j is the price of good j and D_j and S_j are the demand and supply respectively of good j.

This is how prices are determined in the Walrasian case.

Grand utility-possibility curve: The locus of points of Pareto optimum in production and exchange.

Independent of irrelevant alternatives: There is the requirement that social ranking of choices must be independent of irrelevant alternatives. This means two things. Ordinal measure is to be used; interpersonal comparisons of the relative intensity of preferences (cardinal measures) are to be avoided. Secondly , if the group prefers A to B and B to C, then it must continue to prefer A to B whether or not C is present as a possible choice.

Kaldor-Hicks compensation principle: A compensation test of welfare economics stating that there will be net gain in social welfare if those who have welfare gains can both compensate losers and still have a net gain for themselves.

Maximum social welfare: Is attained at the point where grand possibility curve is tangent to a social welfare curve.

Offer Curve : A curve showing those trades an individual would willingly make away from a particular initial endowment at alternative price ratios.

Pareto Optimality : An allocation of resources in which no one individual can be made better off without making someone else worse off.

Partial Vs. General Equilibrium: In partial equilibrium analysis, we study specific decision- making units and markets by abstracting from the interconnections that exist between them and the rest of the economy. The justification for doing this is that partial equilibrium analysis reduces the problem under study to manageable proportions, while at the same time giving us, in most instances, a sufficiently close approximation to the results sought.

In contrast, general equilibrium analysis examines the interrelations among the various decision-making units and the various markets in the economy in an attempt to give a complete , explicit and simultaneous answer to the basic economic questions of what, how and for whom.

The entire economy is in general equilibrium when each decision-making unit and market in the economy is individually and simultaneously in equilibrium.

Can an economy ever reach general equilibrium in the real world? Since in the real world, tastes, technology and the supply of labour and capital are continuously changing, the economy will always be gravitating toward a general equilibrium point, never quite realizing it. In general equilibrium model, everything affects everything else for an economy such as ours economy, composed of hundreds of factors, thousands of commodities , millions of firms and a heft of consuming units. The simple general equilibrium model does show the interrelations between various sectors of the system, however, and gives at least a flavor of truly general equilibrium analysis].

Requirements that is to be fulfilled in Arrow's social choice theory:

The first requirement is that of *collective rationality*. These have three components: Unrestricted scope, transitivity and connectedness. The first one means that the process of aggregation that is chosen must not only leave individual free to express their true preferences but also be able to deal with any set of preference schedule. Transitivity means that, for the collective choice, if alternative A is preferred to B, and B is preferred to C, then A is must be preferred to C. Connectedness means that, for any two items A and B, it must be true either that A is preferred to B or B is preferred to A or one is indifferent to both A and B. Collective rationality, therefore, simply requires that individuals must be free to reveal their real choices and that the process of social choice must be logically coherent.

The send requirement is that social choice must abide by the *Pareto principle*, that is to say that social choices must be responsive to individual choices.

The third condition is that of *non-dictatorship*. Social choice must be true collective choice, the result of aggregating of all individuals' preferences. It must not simply be the expression of one person's will.

Finally, there is the requirement that social ranking of choices must be *independent of irrelevant alternatives*. This means two things. Ordinal measure is to be used; interpersonal comparisons of the relative intensity of preferences (cardinal measures) are to be avoided. Secondly, if the group prefers A to B and B to C, then it must continue to prefer A to B whether or not C is present as a possible choice.

With the above four conditions, Arrow proves that no mechanism for making social choices will always meet all four conditions simultaneously.

At the time Social *Choice and Individual Values was published*, the logic of group decision-making was not even recognized as an economic issue. Since then there has been an overwhelming blossoming of the social choice field.

Scitovsky reversal test: This is a development of the Kaldor-Hicks compensation test which enlarges it by the condition that there is no increase in social welfare by a return to the original situation on the part of the losers.

Several necessary marginal conditions for economic efficiency: The first necessary condition states that the marginal rate of substitution between two commodities should be the same for all consumers. Whenever this is not the case, a transfer of some of these commodities between the two individuals can increase the utilities of both.

The second necessary condition for economic efficiency states that the the marginal rate of transformation between two products should be the same for all producers.

The third necessary condition for economic efficiency states that the marginal physical productivities of any factor in a specific product should be the same for all producers.

The fourth necessary condition for economic efficiency states that the marginal rate of technical substitution between any two factors must be the same for all producers.

The fifth necessary condition for economic efficiency states that the marginal rates of substitution of two products for consumers must equal the marginal rate of transformation of the same two products for firms.

These necessary conditions are all derivable from a single marginal rule. We know that perfectly competitive market leads to the fulfilment of all the necessary conditions for economic efficiency. All these necessary conditions do not constitute a set of sufficient conditions for maximising the value of the social welfare function—or even for achieving economic efficiency.

Theory of social choice: The theory of social choice addresses the problem of how collective choice, i.e., a decision made for the group as a whole might be derived from the preferences of the individuals who constitute the group This issue is of crucial importance to those who investigate human decision making such as economists, political scientists and philosophers.

Welfare economics: The branch of economics that sets out the rules for maximising social welfare of society by considering both the size of social welfare and its distribution. The subject has advanced much from Pigou's *Economics of Welfare*(1919). Though many attempts are done in this field to increase social welfare through redistribution, the first rigorous attempt was done by Kaldor-Hicks compensation principle, the Scitovsky reversal test and Rawl's maximum principle. Other approaches include the attempt to devise a social welfare function , sunk by Arrow's impossibility theorem and the general theory of second best. Amartya Sens crique on Arrow deserves attention in this context.

5.11 Exercises

A. Short-answer Type Questions

1. What is partial equilibrium analysis? Why is it used? What does general equilibrium analysis accomplish? When can we say that the entire economy is in general equilibrium?

2. Why do you need to develop a general equilibrium model in contrast to partial equilibrium model ?

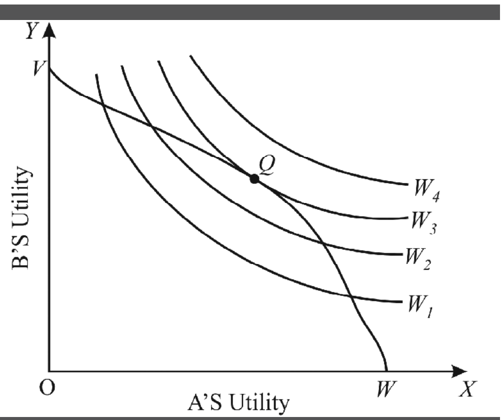
3. What is Walras Law? What are the problems with Walrasian equilibrium?
4. What is an Offer Curve?
5. What is Arrow's Impossibility Theorem? Who criticized Arrow's theorem?
6. Can an economy ever reach general equilibrium in the real world?
7. What is Samuelson's Compensation Principle?
8. State the Second Fundamental Theorem of Welfare Economics.
9. State any <u>two</u> criticisms of Kaldor Hicks in terms of community indifference curve.
10. What is the theory of social choice?
11. What is meant by independent of irrelevant alternatives? Give examples.
12. What is the significance of the Arrow theorem?

B. Medium-answer Questions carrying 5 Marks

1. How are equilibrium prices set in a Walrasian equilibrium?
2. Compare the First and Second Fundamental Welfare Theorems?
3. Explain briefly any two :
   a) A.K. Sen's Capability Approach, b) Scitovsky's double criterion, c) Kaldor-Hicks criteria
4. Discuss the different price mechanisms through which equilibrium price is attained under general equilibrium.
5. What is compensation principle? State some major principles of welfare economics.
6. Various mechanisms for reaching the Walrasian Equilibrium (WE) have been proposed. What are they? What are the problems with the concept of WE?
7. Using the excess demand criteria, explain the different cases under existence of equilibrium.

8. Diagrammatically explain the point of constrained bliss.

[**Hints :**



C. Long-answer Type Questions

1. Write short notes on :
   - Bergsonian Frontier
   - Arrow's Social Choice
2. Critically discuss the 2x2x2 general equilibrium model to derive solutions for
   - The allocation of the given K and L to the production of each commodity (K_x, K_y, L_x, L_y).
   - The quantities of X and Y which will be bought by the two consumers (X_A, X_B, Y_A, Y_B).
   - The prices of commodities (P_x and P_y) and of the factors of production (wage w, and rental of capital r).
3. Critically evaluate compensation criteria. Do they serve any purpose in welfare analysis? Explain.
4. Welfare economics can be interpreted as the economics of efficiency. Several necessary marginal conditions for economic efficiency can be derived, no doubt. State only the necessary conditions.
5. Critically illustrate Arrow's Impossibility Theorem

or

What are the requirements that must be fulfilled in Arrow's social choice theory?

5.12 References

Arrow, K.J (1951) *Social Choice and Individual Values*. New York: Wiley.

Berrgson,A. (1938). A Reformulation of Certain Aspects of Welfare Economics, QJE Vol 47 pp22-29.

Chakravarty, S. R. (2002). *Microeconomics*. Allied Publishers Pvt. Ltd..

Johnson, H. G. (2017). *General Equilibrium Analysis: A Micro-Economic Text*. Routledge.

Kaldor,N. (1939) Welfare propositions of economics and interpersonal comparison of utility. *Economic Journal* 49:549-52.

Hicks, J.R. (1939).Foundations of welfare economics, *Economic Journal* 49: 696—712.

Levin, Jonathan (2006). *General Equilibrium*. Retrieved from https://web.stanford.edu/~jdlevin /Econ%20202/General%20Equilibrium.pdf

Little, I.M.D.(1957). *A Critique of Welfare Economics*, Oxford: Oxford University Press.

McKenzie, L. W. (1989). General equilibrium. In *General Equilibrium* (pp. 1-35). Palgrave Macmillan, London.

Mishan,E.J. (1981). *Introduction to Normative Economics*, New York; Oxford University Press.

Sen, Amartya (1982). *Choice , Welfare, and Measurement*. Oxford: Blackwell.

Sen, Anindya (1999). *Microeconomics, Theory and Applications*. Oxford University Press.

Quirk,James and Rubin Saposnik(1968): Introduction to General Equilibrium Theory and Welfare Economics, New York: McGraw Hill.

Zame,W.(2008). General equilibrium (new developments). *The New Palgrave Dictionary of Economics: Volume 1(8)*, 2411-2425.

Unit 6 ❑ Economics of Risk and Uncertainty

Structure

6.1 Objectives

After going through this unit you will be able

- Learn Von Neumann- Morgenstern Utility Function;
- Have a complete knowledge the problem of choice in situations involving risk;
- Get an idea about expected utility theorem— along with its applications and its critique
- Learn how to measure absolute and relative risk aversion;
- Understand how models of asymmetric information can be used to describe situations where an economic agent has some information which other economic agent do not ;and

- Know the difference between adverse selection and moral hazard problems.

6.2 Introduction

We will discuss in this unit how to model uncertainty and then turn to problems of information. Two types of situation are possible in this context when there is some uncertainty. First, everyone may face the same uncertainty and have the same information available to resolve this uncertainty. In the morning , if there is a very bad information about COVID-19, everyone understand that there is a possibility of its spread. You can ring up the Health Department to get the precise information about the likely state of the COVID pandemic. Secondly, different people may have different information about the same activity or event and this creates some uncertainty. Someone has equipped with better information, someone less. This differential access to information creates uncertainty and we will address this problem in this unit.

6.3 Von Neumann- Morgenstern (VNM) Utility Function

Nicholas Bernoulli in 1728 was the first to hint about the expected utility. But his theory is mostly a descriptive model since he addressed neither the issue of how to measure utility nor that of why his expectation principle would be rational.

But the expected utility theory, developed by von Neumann- Morgenstern, explicitly uses probabilities to help reach in decision under uncertain situations. Their utility function is an elementary utility function v (.) measuring the desirability of the different possible consequences and it is called vNM utility function.

von Neumann- Morgenstern (1944) showed that under several sensible axioms (which they derived), expected utility maximization can be proved to be a rational decision criteria.

6.4 Preferences over Lotteries

The problem of choice in situations involving risk

The traditional theory of consumer behavior does not include an analysis of uncertain situations. Von Neumann and Morgenstern showed that under some circumstances it is possible to construct a set of numbers for a particular consumer that can be used to predict her choices in uncertain situations. In other words, von Neumann- Morgenstern utility theory asserts that a few axioms about an individual's choice would guarantee the existence of a utility function yielding numerical utilities for different outcomes so that the expected utilities of different lotteries, when computed using these utility numbers, would preserve the individual's *preference over lotteries***.** Great controversy has centered around the question of whether the resulting utility index is ordinal or cardinal. It will be shown that von Neumann-Morgenstern utilities possess at least some cardinal properties.

Let us take an example. All automobiles of the same model and produced in the same factory do not always have the same performance characteristics. As a result of random accidents in the production process some substandard automobiles are occasionally produced and sold. The consumer has no way of knowing ahead of time whether the particular automobile which she purchases is of standard quality or not. Let A represent the situation in which the consumer possesses a satisfactory automobile, B a situation in which she possesses no automobile, and C one in which she possesses a substandard automobile. Assume that the consumer prefers A to B and B to C. Present her with a choice between two alternatives: (1) She can maintain the status quo and have no car at all. This is a choice with certain outcome; i.e., the probability of the outcome equals unity* (2) She can obtain a lottery ticket with a chance of winning either a satisfactory automobile (alternative A) or an unsatisfactory one (alternative C). The consumer may prefer to retain her income (or money) with certainty, or she may prefer the lottery ticket with dubious outcome, or she may be indifferent between them. Her decision will depend upon the chances of winning or losing in this particular lottery. If the probability of C is very high, she might prefer to retain her money with certainty; if the probability of A is very high, she might prefer the lottery ticket. The triplet of numbers (P, A, B) is used to denote a lottery offering outcome A with probability $0<P<1$, and outcome B with probability $1 - P$.

Constructing Von Neumann - Morgenstern utility index using the axioms

The Axioms

It is possible to construct a utility index which can be used to predict choice in uncertain situations if the consumer conforms to the following five axioms:

Complete-ordering axiom: For the two alternatives A and B one of the following must be true: the consumer prefers A to B, she prefers B to A, or she is indifferent between them. The consumer's evaluation of alternatives is transitive, i.e., if she prefers A to B and B to C, she prefers A to C.

Continuity axiom: Assume that A is preferred to B and B to C. The axiom asserts that there exists some probability P, 0 < P < 1, such that the consumer is indifferent between outcome B with certainty and a lottery ticket L= (P, A, C).

Independence axiom Assume that the consumer is indifferent between A and B and that C is any outcome whatever. If one lottery ticket L_1 offers outcomes A and C with probabilities P and 1 - P respectively, and another lottery ticket L_2 offers outcomes B and C with the same probabilities P and 1 - P, the consumer is indifferent between the two lottery tickets L_1 and L_2 . Accordingly, if she prefers A to B, she will prefer L_1 to L_2.

Unequal-probability axiom Assume that the consumer prefers A to B. Let L_1 = (P_1,A, B) and L_2 = (P_2,A, B). The consumer will prefer L_2 to L_1 if and only if $P_2>P_1$.

Compound-lottery axiom Let L_1 = (P_1, A, B), and L_2 = (P_2, L_3, L_4), where L_3 = (P_3, A, B) and L_4 = (P_4, A, B), be a compound lottery in which the prizes are Lottery tickets, L_2 is equivalent to L_1 if $P_1 = P_2P_3 + (1-P_2)P_4$. Given L_2 the probability of obtaining L_3 is P_2. Consequently, the probability of obtaining A through L_2 is P_2P_3. Similarly, the probability of obtaining L_4 is (I-P_2), and the probability of obtaining A through L_4 is $(1-P_2)P_4$.

The axioms have been developed for situations in which there are only two outcomes. Assuming that the pair-wise axioms hold, analysis is easily extended to cases *with any number of outcomes.* Let

$$L = (P\ldots\ldots P_n, A_1,\ldots,A_n)$$

denote a lottery with *n* outcomes where 0 < P_i < 1 is the probability of outcome A, and

$$\sum_{i=1}^{n}, P_i = 1.$$

For a consumer who conforms to the five Von Neumann and Morgenstern axioms, cardinal utility index can be constructed as follows: Assume a decision maker has confronted with a chance prospect: He may receive Re 0 (outcome A) with probability 0.50 or Rs, 1000 (outcome B) with probality0.50.Arbitraly aero utile is assigned to the gain of Re 0 and 100utiles to the gain of Rs.1,000 It is thus a gamble (say, L_1) with outcomes A and B of gaining Re 0 and Rs. 1000. The expected value of this gamble L_1 (P,A,B) is

$$E\,[U(L_1)] = PU(A) + (1-P)U\,(B)$$

But, here P= 0.50, and

$$U(A)= U(0)=0$$

$$U(B)= U\,(1{,}000)=100$$

Therefore, $E\,[U(L_1)] = 0.50\,(0)+0.50\,(1oo)=50$.

The *expected utility theorem* can now be stated as follows: for a decision maker whose preferences are consistent with the axioms of complete ordering, continuity, independence, unequal probability, compound lottery, there exists (a) a unique subjective probability distribution for the outcomes of risky choice alternatives, and (b) a utility function U which gives a single utility index, measuring the attractiveness of each risky alternative.

Example: The vNM utility function of an individual is $v = \sqrt{m}$, and her initial wealth is 36. Will she accept a gamble in which she wins 13 with a probability 2/3 and lose 11 with probability 1/3?

[**Ans:** The expected utility of the gamble is given by U=(2/3). $\sqrt{(36+13)} + (1/3)\sqrt{(56-11)}$ = (2/3)7 + (1/3).5 = 19/3 which is $> \sqrt{36} = 6$. Hence the gamble should be accepted.]

Example: Pranab's Von Neumann -Morgenstern utility function can be represented by

$$U = 9+M—0.01M^2$$

Where U is utility and M is monetary gain (in thousands of dollars). He has the opportunity to invest $20,000 in Mathut Finance Company. He believes that there is a two-thirds chance that he will lose his entire investment and a one-third chance that he will gain$ 30,000. (1) If he makes the investment, what is his expected utility? (2) Should he make the investment?

[**Ans:** The utility of no monetary gain (i.e, M=0) equals 9.

The utility of a 20 thousand dollar loss is –9.

The utility of a 30 thousand dollar gain is 30.

If he makes the investment, the expected utility equals

$$2/3\ (-15)+1/3\ (30) = 0$$

(2) Since the expected utility if he makes the investment (i.e., **0**) is less than the expected utility if he does not make it (i.e.,9), he should not make the investment.**]**

6.5 Expected utility theorem— Applications and its critique

Schoemaker (1982) had articulated that the expected utility theory (EUT), supposed to be the major paradigm in decision making since the Second World War, has been used in management science (especially decision analysis), in finance and economics, descriptively by psychologists, and has played a central role in theories of measurable utility. The history of development of the EUT is quite interesting.

Expected Utility

Let us assume that a utility index exists which conforms to the above five axioms. The *expected utility* for the two-outcome lottery L = (P, A, B) is

$$\mathbf{E[U(L)] = PU(A) + (1 - P)U(B)}$$

Consider the lotteries $L_1 = (P_1, A_1, A_2)$ and $L_2 = (P_2, A_3, A_4)$. An expected utility theorem states that if L is preferred to L_2, $E[U(L_1)] > E[U(L_2)$. The significance of this theorem is that uncertain situations can be analyzed in terms of the maximization of expected utility.

Proof of the Theorem

The proof of the theorem is straightforward. Select outcomes such that B. the best, is preferred to all other outcomes under consideration, and W, the worst, is inferior to all other outcomes under consideration. By the continuity axiom Q_i's exist such that A, is indifferent to (Q_i, B, W) (i = 1,... ,4). Thus L_1 and L_2 are equivalent to, i.e., have the same expected utilities as, the lotteries (Z_1, B, W) and (Z_2, B, W) respectively where $Z_1 = P_1Q_1 + (1 - P_1)Q_2$ *and* $Z_2=P_2Q_3 + (1 — P_2)Q_4$. By assumption L_1 is preferred to L_2, and it follows from the unequal probability axiom that $Z_1 > Z_2$. Since origin and unit of measure are arbitrary for utility indexes, let U(B) = 1 and U (W) = 0. Now, $E[U(L_1)] = Z_1$ and $E[U(L_2)] = Z_2$, establishing the theorem.

The expected utility formula may be used to construct utility numbers for a person who conforms to the von Neumann-Morgenstern axioms. Arbitrarily assign utility numbers to two certain outcomes A_1 and A_2. For example, if A_2 is preferred to A_1, let $U(A_1,) = 20$ and $U(A_2)=1000$. Now consider the outcome A_3. If A_3 lies between A_1 and A_2 in the preference ranking, ask the consumer for a value of P such that she is indifferent between A_3 and (P, A_1, A_2). If P = 0.8, solve

$$\mathbf{U(A_3) = 0.8U(A_1) + 0.2U(A_2) = 216}$$

If A_4 is preferred to all three alternatives, its utility can be obtained by asking the consumer for a value of P such that she is indifferent between A_2 and (P,A_1, A_4). If P =0.6, solve

$$1000 = (0.6)(20) + 0.4\ U(A)$$

for $U(A_4) = 2470$. The process can be continued indefinitely, and will not lead to contradictory results as long as the five axioms are obeyed.

The utilities in the von Neumann-Morgenstern analysis are cardinal in a restricted sense. They are derived from the consumer’s risk behavior and are valid for predicting her choices as long as she maximizes expected utility. Von Neumann-Morgenstern utilities possess some, but not all, the properties of cardinal measures. If $U(A) = kU(B)$, it is not meaningful to assert that the consumer prefers A *k* times as much as B. Utility ratios are not invariant under linear trans-formations.

Advantages of EUT

There are many advantages of expected utility theorem:

(1) One advantage is its *analytic simplicity*. To compare two lotteries, one has just to compare the means, that is, the expected values, of the two probability distributions. This simplicity accounts for its pervasive use in economics. It is easy to work with, and difficult to do without, expected utility.

(2) The second advantage is *normative*. Expected utility may provide a valuable guide to action. For example, people often find it hard to think systematically about risk alternatives. But if an individual believes that his choice should satisfy the axioms on which the theorem is based (notably, the independence axiom), the theorem can be used as a guide in his decision- making process.

(3) A third feature is that it is not required that outcomes of lotteries be expressed in nominal values. One outcome may be a consumption bundle, another monetary pay-off, and so on. By assigning utilities, that is numbers, EUT transforms different types of outcomes onto the real line—the real line on which the different lotteries (that is, probability distribution) can be redefined. What the EUT does is to rank these lotteries by considering their first moment (that is , expectation) only. Thus the most specific implication of the EUT from its form or *the preference function* which is linear in the probabilities. Once the origin and the unit of utility scale is chosen, the utility numbers of different outcomes get fixed and hence the expected utility of a lottery is linear.

Example: Suppose your utility function is $1—e^{-w/10000}$, where $e \approx 2.7183$ and w denotes wealth expressed in dollars. Your current wealth is \$5000. You want to maximize your expected utility. Should you take a gamble in which you win \$5000 with probability 0.6 and lose \$4000 with probability 0.4?

[Ans: If you reject the gamble, your utility is $1—e^{-0.5}$ = 1—0.6065=0.3935. If you accept the gamble, your expected utility is $0.6(1—e^{-1}$ +$0.4(1\text{-}e^{-0.1})$ =0.6(0.6321)+0.4 (0.0952) =0.4173. To maximize expected utility, you accept the gamble.]

6.5.1 Applications of Expected Utility and its Critique

Schoemaker (1982) articulated that the expected utility theory (EUT), supposed to be the major paradigm in decision making since the Second World War, has been applied in management science (especially in decision analysis), in finance and economics, descriptively by psychologists, and has played a central role in theories of measurable utility. In addition, this theory is very helpful for understanding the concepts of risk and risk aversion, for example, Arrow –Pratt measures of absolute and relative risk aversion. It has also been used by Mas-Collel (1995) and others to define stochastic dominance of one probability distribution over another.

Perhaps the most widespread use of this theory is in portfolio analysis and capital asset pricing model, which investigates how an individual, faced with the problem of current and future consumption over a horizon of, say T periods (that is $C_0, C_1, \ldots, C_{T-1}$), would put his investible financial wealth into a given number of securities in each period so as to maximize discounted present value of expected utility, conditional on information at time zero. The conventional method to solve such dynamic decision problems under uncertainty is one of the stochastic dynamic programming. The rational individual in this situation must choose consumption such that, along an optimal path, small reallocation of wealth between consumption and investment in any asset will not alter the value of the programme. This EUT is used in the theory of growth and fluctuations. For example, the stochastic version of models of growth and fluctuations is another area where EUT has been used extensively.

Extensions and applications of this expected utility theory have been made to the analysis of insurance and labour markets focusing on issues of signaling, screening and adverse selection, as well as principal –agent problems such as moral hazard.

Critique of the EUT

As a descriptive theory the EUT, in particular its *Independence Axiom* has always been questioned. It has faced a major challenge, associated with the failure of the so-called "independence" axiom to hold. This version of the Allais paradox and more recently the related Machira paradox developed by Cook, K.S. and Margaret Levy (1990) have led to challenge to the conventional neoclassical assumption that economic agents act rationally.

A growing body of empirical evidence exists which reveals patterns of choice behaviour inconsistent with the EUT or Independence Axiom. One of the earliest example of this is the well-known Alliais Paradox which consists of choosing a lottery from each of the two pairs (e_1, p) and (r, s), where each lottery is defined on the tree outcomes: 0.1 million dollars, and 5 million dollars. These lotteries are described in Table below (in which we have also included two others lotteries, q and e_0).

Allais Paradox

Outcome (in million dollars)	**Lottery**					
	e_1	**p**	**r**	**s**	**q**	**e_0**
x_0 **: 0**	0	.01	.90	.89	1/11	1
x_1 **: 1**	1	.89	0	.11	0	0
x_2 **: 5**	0	.10	.10	0	10/11	0

Researchers have given this problem to several hundred people and the modal preference has been found to be for e_1 for the first pair and r in the second pair. But this behavior contradicts both the EUT and the Independence axiom.

There are other types of evidence. Researchers have responded to this growing body of evidence by developing *nonlinear (not-expected utility*) functional forms for individual preference functions or lotteries.

The one of the criticisms of EUT that the independence axiom may be violated systematically, which is referred to as the Allais paradox. Consider for example, a bet, which gives $120.00 with probability 0.9 and $ 0 with 0.1, and a sure receipt of $ 100.00. The typical choice is to take the sure receipt. Now consider two bets, one gives $120.00 with probability 0.45 and $ 0 with probability 0.55, the other gives $ 100 with probability 0.5 and $0 with 0.5. Here the typical choice is to take the first bet. This violates independence.

Second two bets are made by mixing the first two with the lottery that gives $ 0 for sure, with even proportion. One explanation of this is called certainty effect.

Another criticism is that risk attitudes may depend on staus quo points, where the theory assumes that only the distribution over the outcomes matter

Finally, we should also point out that some economists have some reservations against using EUT. They prefer using an ordinal utility function defined over mean and standard deviation (and also higher moments) of the distribution of returns of a security/ portfolio. This in fact belongs to the mean-variance approach to portfolio analysis.

In what follows is that despite a plethora of development and applications of expected utility theory, it has faced several major challenges.

One of the Sums on Expected Utility is given below:

Example : Anamika is planning a trip. The utility from her upcoming vacation is primarily a function of how much money she spends on it,, given by $U(Y) = (Y+625)^{1/2}$, where Y represents the amount of money she spends. Anamika has $5,000 to spend on this trip. If there is a 10% probability that Anamika will lose all her vacation, what the trip's expected utility?

[**Ans:** If Anamika spends all $ 5,000, her utility will be $(5,000+625)^{1/2}$, or 75. If there is a 10% probability that Anamika will lose all her cash, the trip's expected utility is

$$0.10(625)^{1/2} + 0.90\ (5,625)^{1/2} = 70.]$$

6.6 Attitude towards Risk

This section will deal with consumer behavior under uncertainty.

It is now assumed that the utility function: (1) has the single argument "wealth" measured in monetary units, (2) is strictly increasing, and (3) is continuous with continuous first- and second-order derivatives.

Attitudes toward Risk

The expected value of the lottery (P, W_1, W_2), where the W_i are different wealth levels, is the sum of the outcomes, each multiplied by its probability of occurrence

$$E[W] = PW_1 + (1 - P)W_2$$

A person is *risk neutral* relative to a lottery if the utility of the expected value of the lottery equals the expected utility of the lottery, i.e., if

$$U[PW_1 + (1 - P)W_2] = PU(W_1) + (1 - P)\ U(W_2)$$

Such a person is only interested in expected values and is totally oblivi-ous to risk. If she is risk neutral toward all lotteries, the immediate above equation implies that she has a linear utility function of the form $U = \alpha + \beta W$ with $\beta > 0$.

A person is a *risk averter* relative to a lottery if the utility of its expected value is greater than the expected value of its utility:

$$U[PW_1 + (1 - P)W_2] > PU(W_1) + (1 - P)\ U(W_2)$$

Such a person prefers a certain outcome to an uncertain one with the same expected value. If above inequality holds for all $0 < P < 1$ and all and W_2 within the domain of the utility function, the utility function is strictly concave over its domain If $d^2U/dW^2 < 0$, the utility function is strictly concave and the consumer is a risk averter.

Introspection and observed behavior suggest that most people are risk averse in most of their dealings. Nonetheless, the analysis can cover equally well a person who prefers uncertain outcomes. A person is a *risk lover* relative to a lottery if the utility of its expected value is less than its expected utility. In this case the inequality of above is reversed. A risk lover will always take a fair bet (i.e., one in which the expected value of the gain equals the expected value of the loss). Following the argument used for a risk averter, if $d^2U/dW^2 > 0$, the utility function is strictly convex and the consumer is a risk lover.

It is possible for a person to be a risk averter in some situations and a risk lover in others. Consider, for example, a low income person who is risk averse in almost all of her dealings except that she will pay a dollar for a lottery ticket with an expected value of 50 cents. Her behaviour would be consistent if her utility function had the shape depicted in Fig. 6.1. W_1 is her wealth position if she loses the lottery, and W_2 is her position if she wins. Her utility function is strictly concave for $0 \leq W \leq W_0$ and strictly convex for $W > W_0$. Consequently, she is risk averse for all uncertain situations in which the best outcome is no greater than W_o.

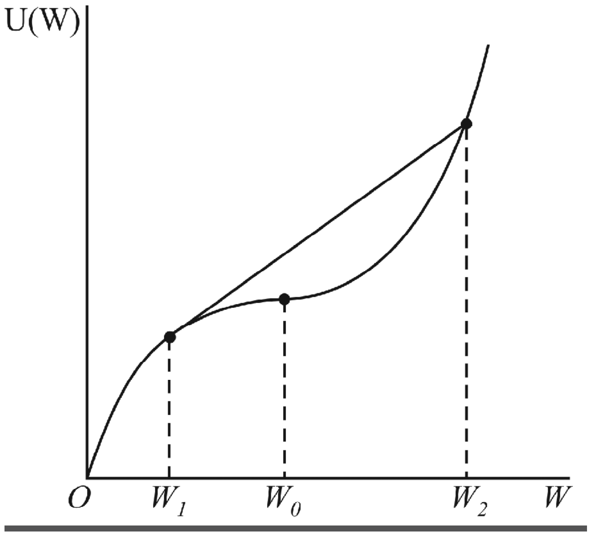


Fig: 6.1

The sign of the second derivative of the utility function provides an indication of the consumer's attitude, but since its magnitude is not invariant under a linear transformation, it cannot be used to indicate the level of risk aversion or preference.

6.6.1 Measures of absolute and relative risk aversion (Arrow – Pratt)

It is sometimes important to know how *averse to risk* a certain individual is. To this effect there are a set of tools to measure risk in a quantitative way. The most common and frequently used measure of risk aversion is the Arrow-Pratt measure of absolute and relative risk aversion. Named after John W. Pratt's paper "Risk Aversion in the small and in the Large", 1964, and Kenneth Arrow's "The Theory of Risk Aversion", 1965, these are the measures**:**

Arrow-Pratt measures of absolute and relative risk-aversion.:

A measure of ***absolute risk aversion,*** *r,* is provided by the ratio of the second and first derivatives of the utility function.

$$r = -\frac{U''(W)}{U'(W)} = -\frac{d \ln U'(W)}{dW}$$

This measure is positive, negative, or zero as the consumer averts, prefers, or is neutral toward risk.

Consider a quadratic utility function, $U = W—aW^2$ with $a>0$ and the domain $0 < W < 1/(2a)$. It describes risk-averse behavior since $U" = -2a<0$. Evaluating the above equation, we get

$$r = \frac{2\alpha}{1-2\alpha W}$$

with $dr/dW = 4\alpha^2 / (1-2\alpha W)^2 > 0$. Here risk aversion increases with wealth.

This utility function has several attractive feature—it has linear marginal utility (useful for solving dynamic buffer stock problems) and it allows expected utility to be expressed in terms of the mean and variance of income alone.

Most often one would assume that the opposite is the case. The utility function $U = \ln(W + a)$ with $a > 0$ exhibits decreasing risk aversion.

The utility function with *constant absolute risk aversion* is.

$$U(W) = -ke^{-AW}$$

or, $$-\frac{U''(W)}{U'(W)} = A$$

This class is referred to as the exponential utility function.

Arrow –Prat measure of relative risk aversion is

$$r' = -W\frac{U''(W)}{U'(W)}$$

where W is the payoff of a given lottery and U(w) the utility derived from that payoff.

Here are the several examples:

Example: Suppose that an individual's utility function is v (w) =w^2.Then v´´ (w) = 2> 0, and the persons is a risk lover. Will the person accept the gamble where there is a 50% chance of winning or losing 20%, if the person's initial wealth is 100? What would happen if the person is risk neutral?

[**Ans:** If the person does not accept the gamble, then utility is 100^2 = 10,000. If the gamble is accepted, the expected utility is 0.5(120^2) + 0.5 (80^2) = 0.5(14400 + 6400) = 10,400. Hence the gamble is accepted.

It can be shown that if v=w, the person is risk neutral and will be indifferent between accepting and rejecting the gamble.]

Example: For any utility function U (M), absolute risk aversion R is defined as

$$R = -\frac{U''(M)}{U'(M)}$$

where U´ (M) and U´´ (M) are the first and second derivatives of U, respectively. This measure is positive, negative, or zero as the consumer avoids, prefers, or is neutral toward risk.

What can you say about the consumer with the following utility functions?

(1) U(M)= a+bM, a, b>0

(2) U(M)= Log (a+bM) m, a,b>0

(3) U(M)= aM+ bM^2

[**Ans** (1) U´(M) = b, U´´(M) = 0, R = 0.

The consumer is risk neutral

(2) U´ (M)= b/ (a+bM), U´´(M)= –b/ (a+bM^2), and R= b/(a´´bM2) >0.

The consumer is risk averse.

(3) U´(M)= a +2bM , U´´(M)=2b, and R=—2b/(a+2bM) <0.

The consumer is a risk lover.]

Example: An important question is whether an individual's degree of aversion to risk increases or decreases for higher levels of wealth. How can you justify the statement with examples?

[Ans: If the utility function is quadratic in wealth, we can write:

$U(W)= a+ bW+ cW^2$ (where $b>0,c<0$), Pratt's risk aversion function is

$$r(W) = -\frac{U''(W)}{U'(W)} = \frac{-2c}{b+2cW}$$

which increases as wealth increases.

On the other hand, if utility is logarithmic in wealth,

$U(W)= \log (W+a)$ $a>0$ we have

$$r(W) = -\frac{U''(W)}{U'(W)} = \frac{1}{W+a}$$ which does decrease as wealth increases.

6.7 Adverse Selection and Moral Hazard

The problems with which we are interested in this section come under the broad heading of *asymmetric information models.* These models describe economic interactions between two economic agents or two players in a game. In some games with asymmetric information, one player will be a seller, and other player will be a buyer, though the exact characteristics of either player will typically depend on the particular example in question. One player knows something that the other does not know. The baker knows the quality of the loaf of bread being sold, but the customer in the bakery does not.

These asymmetric information models are broken down into two general categories:

1. In a model with *adverse selection problem*, one player knows some piece of information or *type*, but the other player does not. This type is determined by nature, and cannot be affected by either player. Adverse selection problems involve a *hidden type*.

The notion of adverse selection is applied widely because asymmetric information is often a common feature of market interactions. This typically occurs when the private information available to the sellers is not disclosed to the buyers or vice versa. The most classic example of adverse selection concerns the market for secondhand cars, which is referred to as the "market for lemons" since the pioneering work in 1970 of the Nobel Laureate Economist G.A. Akerlof.

In that market potential buyer cannot distinguish good cars (" peaches") from bad cars (" lemons") easily.

However, the sellers are perfectly aware of the characteristics of their cars. In that process the resulting market equilibrium will be inefficient because there will be number of transactions that are lower than the optimum level. Here the salesman knows whether the car is a high-quality or a low-quality vehicle. But whether the car is lemon or not was decided by *nature*, not by the salesman. The *type* of the car is known by the salesman but is not known by the buyer. This is an adverse selection problem, since it involves a hidden type.

2. In a model with a moral hazard problem, one player can take an *action* which is not observed by the other player. Moral hazard problems involve a *hidden action.**

To continue with the examples, the baker has the ability to choose the level of quality of loaf of bread. This is an example of moral hazard problem. The baker knows the quality of the loaf of bread being sold, but the customer does not. Furthermore, by taking an action, the baker can decide whether the loaf is a high-quality loaf or just another regular loaf of bread. The baker takes an action which is hidden from the other player.

* Here is a simple trick to remember the difference between adverse selection and moral hazard problems. The first two letters of "hazard" stand for "hidden action."

6.8 Conclusion

In what follows we have analyzed in detail the behavior of consumers in the backdrop of risk and uncertainty. The approach of von Neumann and Morgenstern in this context is concerned with the consumer's behavior in situations characterized by uncertainty. If the consumer behavior satisfies some crucial axioms, her utility function can be derived by presenting her with a series of choices between a certain outcome on the one hand and the probabilistic combination of two uncertain outcomes on the other. The utility function thus derived is unique up to a linear transformation, and provides a ranking of alternatives in situations that do not involve risk. Consumers maximize expected utility, and von Neumann-Morgenstern utilities are cardinal in the sense that they can be combined to calculate expected utilities and can be used to compare differences in utilities. The expected utility calculations then can be used to determine consumer's choices involving risk.

With wealth as a single argument of the utility function, the utility of the expected value of the outcome of an uncertain situation exceeds the expected utility of the outcome for a risk averse consumer; i.e., her utility function is strictly concave. Risk –Averse consumers will pay a premium for insurance to convert an uncertain outcome into a certain one, for example. Similarly, risk lovers and risk neutrals have strictly convex and linear utility functions respectively.

Last of all we have looked at the problem of information (or the lack thereof) between sellers and buyers.

6.9 Summary

Adverse selection: arises in a market when asymmetric information problems drive out higher –quality goods or services.

Allais paradox: Violation of cancellation axiom, first reported by Maurice Allais in 1953.

Cancellation axiom asserts that if we prefer outcome A to B, then we should prefer a lottery that yields A with probability p and C with probability 1-p to a lottery that yields B with probability p and C with probability 1-p, for any c and any p>0.

Asymmetric information: arises when one party to an transaction has more information about a product than his or her counterpart.

Axioms of Von Neumann and Morgenstern: Three of the axioms are stated below:

The **completeness axiom** asserts that an individual with two options prefers the first to the second, prefers the second to the first, or is indifferent to them.

The **transitivity axiom** states that if an individual prefers option A to option B and option B to option A , the individual prefers option A to option C.

The cancellation axiom : asserts that in choosing between two actions, we can ignore states of the world in which the two actions have the same consequences. The remaining axioms are a little more complicated.

Compound lottery: A lottery with two or more stages, whose outcomes are themselves lotteries except in the final stage.

Difference between an adverse selection and moral hazard: An adverse selection problem exists when the informational asymmetry involves knowledge about a hidden type, while a moral hazard problem involves knowledge about a hidden action.

Expected value: Mean

Expected utility: The average utility from a risky situation. If there are n outcomes, $\mathbf{X_1, \ldots, X_n}$ with probabilities $P_1,\ldots,P_n$ $(\Sigma^n Pi = 1)$, then the expected utility is given by

$$E(U)= P_1U(X_1)+ P_2U (X_2)+\ldots+ P_n (X_n)]$$

The expected utility theory (EUT) is a model that represents preferences over risky objects, by weighted average of utility assigned to each possible outcome, where the weights are the probability of each outcome.

The primary motivation for introducing expected utility is to explain attitudes toward risk. consider, for example, a lottery, which gives $ 100 and $ 0 with even chances, and a sure receipt of $ 50. Here typically one chooses the sure receipt, whereas the two alternatives yield the same expected return. Another example is the Saint Petersburg Paradox.

The theory originates from Daniel Bernoulli (1700-1780), an 18th century mathematician, and , was given an axiomatic foundation by John Von Neumann and Oscar Morgenstern in the 1940s. They started from a preference ranking of probability distributions over outcomes and provided the conditions for its expected utility. The condition consists of three axioms**:** weak order, continuity, and independence. The most prominent axiom is independence.: When the decision makers prefers distribution p to distribution q, then he or she prefers the distribution made by mixing p and any other distribution r with proportion $\lambda : 1–\lambda$, that is, $\lambda p +(1 - \lambda)r$ to the distribution made by mixing q and r with the same proportion, that is $\lambda q+(1 - \lambda)r$

Moral hazard: undesirable or reckless behavior in an economic context where there are no incentives to avoid such behavior.

How to solve a moral Hazard Problem? A classic moral hazard problem exists between a landlord and a tenant. Before she moves into the apartment, a tenant knows whether she is a good tenant or bad tenant. She may be a good tenant in the sense that she takes good care of the apartment and does not have loud parties until 4 p.m which disturbs other tenants. Or she may be bad tenant, who vacuums only once in a year, and plays her T.V very loud at all hours of the night.

The tenant can take an action which determines whether she is a good tenant or bad tenant, but this action is hidden from the landlord. The tenant can decide how often to vacuum her apartment, and when and how loud to play her T.V. The informational asymmetry between the two players involves a hidden action. This is a moral hazard problem.

The above fact has direct implications for the outcome of the game which is played between the tenant and the landlord. If the tenant moves in and the landlord accepts the tenant, a contract is signed between the two parties, which take the form of a rental agreement. This rental agreement is made up of various clauses which are designed to protect both tenant and landlord from the negative effects of any moral hazard problems

Risk aversion: Unwillingness to accept fair bets. Arises when an individual's utility of wealth function is concave (that is, $U'(W) > 0$, $U''(W) < 0$).

Saint Petersbug Paradox: A parable of **game theory** identified by Bernoulli and so-called because of its first being stated in the *Commentarii* of Saint Petersburg Academy of Science in the 1730s. This paradox in a game of chance is that the mathematical expectation of game is infinite but the fair price to the player is finite. Bernoulli's approach to this problem was to replace mathematical expectation (probabilities of winning multiplied by monetary prices) by moral expectation (probabilities of winning multiplied by personal utilities).

von Neumann and Morgenstern's analysis of expected utility: a compelling argument for maximizing expected utility was finally established in 1944 by a mathematician, John von Neumann and an economist, Oscar Morgenstern. They proposed eight axioms to characterize rational choice under risk and demonstrated that the only strategy consistent with these axioms is that of expected utility maximization. Not only is Von Neumann –Morgenstern utility theory is different from what Bernoulli conceptualized, it is more general in the sense that it is applicable to any type of outcomes of gamble, not merely monetary outcomes.

Von Neumann—Morgenstern expected utility analysis focuses that agents maximizes utility under uncertainty. Informational constraints and asymmetries generate uncertainty which is analyzed by generating subjective expected utility functions.

Von Neumann and Morgenstern focused on cases in which probability can be interpreted as a long-run frequency. However, obtaining and processing all the information needed for maximizing expected utility can be difficult. Recent empirical studies suggest that expected utility theory makes poor prediction. For example, Von Neumann and Morgenstern's expected utility theorem rigorously prove that if you want to satisfy some basic rationality axioms then you must maximize expected utility. Kahneman and late Tversky showed people do not choose by maximizing expected utility. To predict choices, 'prospect theory' works much better.

6.10 Exercises

A. Short-answer Type Questions

1. What is the decision-maker under uncertainty about?
2. What is Allais paradox?
3. What form does uncertainty take?
4. What can be done to act upon or improve the information available?
5. What is expected utility?
6. Why in an uncertain environment, every act is a gamble or a lottery?
7. Suppose a consumer is initially unaware of the quality of wine. Here the consumer could use price as a signal of the quality of wine. But another option would be to do some research into different types of wine, by reading magazines about wine, or taking a course on wine. Does an information asymmetry exist

between wine producers and this wine consumer? What type of information problem exists Is it a moral hazard problem?

8. The vNM utility function of an individual is $v = \sqrt{m}$, and her initial wealth is 36. Will she accept a gamble in which she wins 13 with a probability 2/3 and lose 11 with probability 1/3?

9. Suppose that an individual's utility function is $v(w) = w^2$.Then $v''(w) = 2 > 0$, and the persons is a risk lover. Will the person accept the gamble where there is a 50% chance of winning or losing 20%, if the person's initial wealth is 100? What would happen if the person is risk neutral?

10. Suppose your utility function is $1—e^{-w/10000}$, where e ≈ 2.7183 and w denotes wealth expressed in dollars. Your current wealth is $5000. You want to maximize your expected utility. Should you take a gamble in which you win $5000 with probability 0.6 and lose $4000 with probability 0.4?

11. Mention three main axioms that were demonstrated by Von Neumann and Morgenstern?

12. For what purpose the concept of adverse selection is used? Define in this connection the term 'market failure.'

13. The notion of adverse selection is applied widely' –Do you agree. What would be the resulting market equilibrium in this process?

14. Which class of function is referred to as the exponential utility function? In this context define Arrow-Pratt measure of absolute and relative risk aversion.

B. Medium-type Questions

1. For any utility function U (M), absolute risk aversion R is defined as

$$R = -\frac{U''(M)}{U'(M)}$$

where U´ (M) and U´´ (M) are the first and second derivatives of U, respectively. This measure is positive, negative, or zero as the consumer avoids, prefers, or is neutral toward risk.

What can you say about the consumer with the following utility functions?

(i) U(M)= a+bM, a, b>0

(ii) U(M)= Log (a+bM) m, a,b>0

(iii) U(M)= aM+ bM^2

2. Anjana is planning a trip. The utility from her upcoming vacation is primarily a function of how much money she spends on it,, given by U(Y)= $(Y+625)^{1/2,}$, where Y represents the amount of money she spends. Anjana has $5,000 to spend on this trip. If there is a 10% probability that Anjana will lose all her vacation, what the trip's expected utility**?**

3. An important question is whether an individual's degree of aversion to risk increases or decreases for higher levels of wealth. How can you justify the statement with examples?

4. When is a person would be considered as risk lover? What is her unique characteristic? Is it possible for a person to be a risk averter in some situations and a risk lover in others?

5. Write a short note on the concept of adverse selection and moral hazard.

C Long-answer type Questions

1. Srijit's's Von Neumann -Morgenstern utility function can be represented by

$$U=9+M—0.01M^2$$

Where U is utility and M is monetary gain (in thousands of dollars). He has the opportunity to invest $20,000 in Mathut Finance Company. He believes that there is a two-thirds chance that he will lose his entire investment and a one-third chance that he will gain$ 30,000. (1) If he makes the investment, what is his expected utility? (2) Should he make the investment?

2. What is expected utility? What is the primary motivation for introducing expected utility? Give examples. How does expected utility theory resolve the Saint Petersbug paradox? Write down two criticisms to the expected utility theory.

3. What are the essential differences between adverse selection and moral hazard?

   Give an example of moral hazard problem. How can you solve the moral hazard problem from your given example.

6.11 References

Akerlof, G (1970).The market for "Lemons": qualitative uncertainty and the market mechanism. *Quarterly Journal of Economics*,84,pp488-399.

Arrow,K.J (1965).*Aspects of the Theory of Risk- Bearing*. Helsinki: Academic Bookstore.

Baumol, W.J.(1977) *Economic Theory and Operations Analysis,* PHI.

Cowell,F. (2006). *Microeconomics*, Oxford University Press, 2006.

Gans, J., King S, Stonecash R, Mankiw NG (2011). *Principles of economics*, Cengage Learning; 2011

Gibbons, Robert: *Game Theory for applied economists*, 1992, Princeton University Press .

Gravelle,H and R. Rees (2004). *Microeconomics*, Pearson Education, 3 rd Edition.

Henderson James M. and Richard E. Quandt. *Microeconomic Theory: A Mathematical Approach.* 3rd Edition, Mcgraw Hill Book Company 1980

Nicholson,*W. Microeconomic Theory*, The Dryden Press, Japan,1985.

Pratt, J.W. ((1954).Risk aversion in the small and in the large. *Econometrica*,vol32 (Jan-April) pp132-136)

Schoemaker, P.J.H., (1982) The expected utility model: Its variants, purposes, evidence and limitations, *Journal of Economic Literature*,XX,pp.539-63.

Sen , Anindya (1999). *Microeconomics : Theory and Applications* .New York: Oxford University Press.

Silberberg,E,: *The Structure of Economics: A Mathematical Analysis*, 1990, McGraw Hill, Second Edition.

Snyder,C and W. Nicholso (2010) *Fundamentals of Microeconomics,* Cengage Learning (India),

Varian, Hall R. (1992). *Microeconomic Analysis,* W.W. Norton and Company, 3rd Edition,.

Varian. Hall R (2004) : *Intermediate Microeconomics: A Modern Approach* (Eighth International Student Edition). 2014.

Von Neuman,J. and O. Morgenstern: *Theory of Games and Economic Behaviour*. Princeton University Press:1944